Markups, Labor Market Inequality and the Nature of Work*

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Abstract

We develop a framework for understanding the effects of a change in markups on the income distribution. We demonstrate the importance of distinguishing between production and expansionary uses of labor for this question. An increase in markups redistributes earnings away from production labor and toward expansionary labor, and has an ambiguous effect on the overall labor share that depends on the relative importance of production and expansionary activities in the aggregate economy. We measure the production and expansionary content of different occupations from the co-movement of occupational income shares with markup-induced changes in the labor share. We find that around one-fifth of US labor income compensates expansionary activities, and that occupations with larger expansionary content have experienced the fastest wage and employment growth since 1980. Our framework can be applied more generally to study the distributional implications of shocks, policies and secular forces that affect the economy by changing markups.

JEL Codes: D2, D3, D4, E3, E5, J2, L1

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1 Introduction

The wedge between the marginal cost paid to factor inputs and the price of final goods paid by consumers, known as the markup, plays a central role in macroeconomics. In the long-run, markups reflect the nature of competitive forces and are a key channel through which industrial and trade policies affect the economy. In the short-run, movements in markups are the main channel through which demand shocks and monetary policy affect the economy, when viewed through the lens of New Keynesian models of the business cycle. In this paper, we develop a framework for understanding the effects of a change in markups on the income distribution. Most existing work in this area has focused on the division of income between payments to labor versus economic profits and payments to other factors. Instead, our focus is on the way that markups affect the division of labor income across different workers according to their role in the production process. We use our framework to interpret differences in the exposure of different occupations to aggregate fluctuations and to offer a new perspective on the relative wage and employment growth in different occupations.

Our main innovation is to distinguish theoretically and empirically between two uses of labor in a modern economy. We refer to the traditional role of labor as an input to the production of existing goods for sale in existing markets as *production*, or Y-type, labor. We contrast this with an alternative use of labor that facilitates extensive-margin replication, which we refer to as *expansionary*, or N-type, labor. N-type labor encompasses a broad array of corporate activities that include overhead, product design, research and development, logistics, marketing and general management capabilities. We show that incorporating these expansionary uses of labor into an otherwise standard economic setting has important implications for the joint behavior of markups and labor income. Distinguishing between N-type and Y-type activities is essential for understanding how different workers are impacted by shocks and policies that propagate by altering markups. We use our framework to estimate that roughly one-fifth of total US labor income compensates N-type activities, and that those occupations whose share of N-type activities is largest are the same occupations that have experienced the fastest wage and employment growth over the last forty years.

A motivating firm problem We start with a simple single firm problem that demonstrates the difference between the way that markups impact the demand for N-type versus

Y-type labor:

$$\Pi = \max_{L_Y, L_N, p_i, y_i} \int_0^N p_i y_i di - W_N L_N - W_Y L_Y$$

$$\text{subject to } Y = L_Y^{\gamma_Y},\ Y = \int_0^N y_i di,\ N = L_N^{\gamma_N},\ y_i = p_i^{-\sigma}$$

A firm hires Y-type labor, L_Y, to produce output Y according to a production function with elasticity γ_Y, which we refer to as the production elasticity. It can sell its output in different symmetric markets, indexed by i. In each market it faces a constant-elasticity demand curve with common elasticity σ and sets a price p_i. The firm can choose how many markets N it wishes to operate in. The benefit of selling in more markets is that it faces a separate demand curve in each market. The cost of selling in more markets is that to do so it must hire more N-type labor, L_N. The measure of markets that can be serviced with a given amount of N-type labor is determined by a span-of-control function with elasticity γ_N, which we refer to as the expansion elasticity. The firm takes the prices of each type labor (W_Y, W_N) as given. Note that when $\gamma_N = 0$, the firm hires only Y-type labor and the choice of how many markets to operate in disappears. In this case the problem collapses to a text-book single firm decision problem.

Rearranging the first-order conditions from this problem yields expressions for the distribution of the firms' revenues between each type of labor and economic profits:

$$\begin{aligned} S_{LY} &= \tfrac{1}{\mu}\gamma_Y \\ S_{LN} &= \left(1 - \tfrac{1}{\mu}\right)\gamma_N \end{aligned} \qquad S_\Pi = 1 - \gamma_N - (\gamma_Y - \gamma_N)\,\tfrac{1}{\mu}$$

The markup charged by the firm is given by $\mu := \frac{\sigma}{\sigma-1}$ and the price charged and quantity sold is the same in each market, $p_i = p$. In this simple model, changes in the markup arise only due to changes in the demand elasticity, σ, although it will turn out that the forces at work are applicable much more broadly. The shares of revenue that go to $Y-$type labor, $N-$type labor and economic profits are denoted by $S_{LY} := W_Y L_Y / pY$, $S_{LN} := W_N L_N / pY$ and $S_\Pi := 1 - S_{LY} - S_{LN}$, respectively. The overall labor share is given by $S_L = S_{LY} + S_{LN} = \gamma_N + (\gamma_Y - \gamma_N)\,\frac{1}{\mu}$.

These expressions reveal several insights about the relationship between the markup and the distribution of revenue. First, in the standard problem without N-type labor $(\gamma_N = 0)$, the firm's revenues are divided between labor and profits in the familiar proportions $\left(\frac{\gamma_Y}{\mu}, 1 - \frac{\gamma_Y}{\mu}\right)$. In this case, an increase in the markup unambiguously leads to a redistribution of revenues away from workers and towards the owners of the firm. This is the source of the conventional wisdom that higher markups are beneficial to firm owners and costly to workers. Versions of this simple inverse relationship between the labor share, production elasticity and markup, $S_L = \frac{\gamma_Y}{\mu}$, underscore almost all existing strategies for measuring the level and dynamics of markups from production data.

Second, when $\gamma_N > 0$, the firm employs workers for expansionary activities as well as for production activities. For workers performing N-type activities, this conventional wisdom is overturned. An increase in the markup redistributes income towards N-type labor, so that the share of the firm's revenues that compensates N-type activities rises while the share that compensates Y-type activities falls. Thus a change in the markup leads to redistribution *within* labor, in addition to the conventional focus on redistribution *between* labor and profits. Thus to know whether the income share of a given worker is positively or negatively exposed to a change in markups requires knowledge of the extent to which that worker's earnings are compensating Y-type versus N-type activities.

Third, the introduction of N-type labor undoes the tight negative relationship between the markup and the overall labor share. Whether the share of the firm's revenues accruing to labor rises or falls following an increase in the markup depends on whether the gains for N-type labor offset the losses for Y-type labor. These expressions reveal that this will be the case whenever $\gamma_N > \gamma_Y$. In the conventional model with $\gamma_N = 0$, this is never the case. But when $\gamma_N > 0$, which is a necessary condition for the existence of N-type labor, the relationship between markups and the labor share is ambiguous. In Section 3 we will argue that the post-war US economy is best described as one in which not only is $\gamma_N > 0$, but $\gamma_N > \gamma_Y$.

The point of presenting this single firm problem at the outset is to illustrate why acknowledging N-type labor is essential for understanding the relationship between markups and the income distribution. In the remainder of the paper we explore the ramifications and limitations of this very simple insight, and we embed this basic trade-off into a general equilibrium structure amenable for estimation with available data.

Outline The first part of the paper is theoretical. In Section 2, we consider a static general equilibrium version of this firm decision problem in which labor supplied by different occupations are imperfect substitutes in both production and expansion. Because different occupations are used with different intensities in production versus expansion activities, the share of labor income received by each occupation responds differently to a change in markups.

Our theoretical results are summarized in three simple but powerful theorems that describe how a change in the markup redistributes national income. Theorem 1 shows that an increase in the markup unambiguously redistributes income away from Y-type labor and towards N-type labor. Thus when markups change, some workers' incomes rise and some workers' incomes fall, depending on the nature of the work that they perform. Theorem 2 then shows that whether the share of labor income received by a particular occupation rises or falls with markups depends only on the relative size of two occupation-specific parameters that reflect their importance in aggregate production and expansion activities, respectively. Thus learning about whether the earnings of a particular occupation are positively or negatively exposed to a change in markups boils down to measuring the size of these two parameters. Finally, Theorem 3 shows that whether an increase in the markup leads to an increase or a decrease in the overall labor share is theoretically ambiguous and depends on the relative size of the aggregate production and expansion elasticities. Together with the average markup, the relative size of these two elasticities also determines the share of overall labor income in the economy that compensates N-type activities. In existing models that abstract from the expansionary role of labor, the markup and the labor share move in opposite directions. But if a sufficiently large fraction of labor income compensates N-type activities – corresponding to a higher expansion elasticity than production elasticity – this co-movement is reversed.

The second part of the paper is empirical. We quantify how much of US labor income compensates N-type activities and which occupations are the most N-intensive. We estimate the structural parameters that govern these quantities in two steps.

In Section 3 we exploit the prediction of Theorem 3 that the sign and strength of the co-movement of the markup with the labor share is informative about the production and expansion elasticities, and therefore the fraction of labor income that compensates Y-type versus N-type labor. We explain how these aggregate structural parameters can be identified given de-trended time-series data on the labor share and a proxy for the aggregate markup. We explain why, in the context of our model, existing approaches for measuring average markups from production data cannot be applied and why we must seek an alternative empirical approach. We show that in our framework, the ratio of price indexes for different stages of production can be used as a valid proxy for the markup. We estimate the production and expansion elasticities using business-cycle frequency US data and show that they imply that between 5% and 35% of labor income compensates expansionary activities, depending on the assumption we make about the average markup over this period.

In Section 4 we exploit the prediction of Theorem 1 that, given the parameter estimates from the first step, the sign and strength of the co-movement of occupational labor income shares with markup-induced variation in the overall labor share reveals the relative intensity of each occupation in Y-type versus N-type activities. Occupations that are more intensive in expansionary activities are those whose share of aggregate labor income increases when the overall labor share increases due to a change in the markup. We estimate the model parameters using three different approaches to isolate markup-induced variation in the labor share: (i) using the de-trended labor share itself; (ii) using the de-trended markup as an instrument; and (iii) using external estimates of lagged monetary policy shocks as a set of instruments. We clarify the orthogonality conditions that are required for each approach.

Regardless of which of these sources of variation that we exploit, we find that the occupations that are the most N-intensive are those that are typically associated with white-collar jobs (high-tech, service, managerial and admin occupations), while those that are the most Y-intensive are those that are typically associated with blue-collar jobs (construction, extractive, farming, machine operators, production and repair occupations). We find both high-wage and low-wage occupations among the N-intensive occupations, so the correlation of wages with the expansionary content of occupations is weak. But we find a strong positive correlation between the expansionary content of occupations and both wage growth and hours growth over the last 40 years. This suggests that the demand for N-type labor is growing faster than is the demand for Y-type labor.

Additional Implications Although our main contribution is to offer new insights into how the labor income distribution and the aggregate labor share are influenced by factors that affect equilibrium markups, our framework also sheds new light on several issues of relevance to macroeconomics and labor economics. First, we offer a simple explanation for the counter-cyclicality of the labor share, conditional on demand shocks, which has been a long-standing puzzle for New Keynesian models (Cantore et al., 2021). Second, a corollary of the counter-cyclical labor share is that our framework offers a mechanism for generating pro-cyclical profits in response to monetary policy and other aggregate demand shocks in New Keynesian models. This is a problematic feature of existing Heterogeneous Agent New Keynesian (HANK) models in which the counter-cyclicality of profits generates counterfactual patterns of wealth redistribution (McKay et al., 2016; Kaplan et al., 2018; Bhandari et al., 2018). Third, our occupational framework suggests a mechanism for endogenizing the labor income distribution in models with time-varying markups, such as HANK models and endogenous growth models.

Fourth, our measures of the expansionary and production content of different occupations offer an alternative to the task-based framework of Autor et al. (2006) and Acemoglu and Autor (2011) as a lens through which to view changes in the US occupational structure over the last forty years. We correlate our measures of the N-intensity of occupations with the manual, routine and abstract content of occupations as measured by Autor et al. (2006).

We find that N-intensity is weakly negatively correlated with the manual content of occupations and weakly positively correlated with the abstract content, but is not correlated with the routine content. Our model and exercises are also related to several other strands of literature that we discuss at appropriate points in the paper.

2 Theoretical Framework

2.1 Economic Environment

Occupations There are a fixed set of J occupations, indexed by $j = 1 \dots J$. We assume that workers are attached to only one occupation.[1] We denote the total quantity of labor supplied by workers in occupation j as L_j, and the corresponding wage rate per unit of labor as W_j. Firms can hire workers in occupation j to perform either Y-type or N-type activities. We denote the quantity of labor demanded from occupation j to perform each activity as L_{jY} and L_{jN} respectively. Labor market clearing in each occupation implies that

$$L_j = L_{jY} + L_{jN} \ \forall j.$$

Upstream sector A representative upstream firm hires labor from each occupation to perform Y-type activities. It produces a homogeneous intermediate good M that it sells to downstream firms in a competitive market at price P_U. The firm's production function combines the labor of different occupations using a Cobb-Douglas aggregator, and has an overall elasticity with respect to the combined labor input of γ_Y, which we label the production elasticity. The occupation weights satisfy $\eta_{jY} \in [0,1]$, $\sum_{j=1}^{J} \eta_{jY} = 1$ and capture the relative importance of each occupation in Y-type activities. The upstream firm thus chooses labor in each occupation L_{jY} and output M to maximize profits Π_U:

$$\begin{aligned} \Pi_U := & \max_{M, \left\{L_{jY}\right\}_{j=1}^{J}} P_U M - \sum_{j=1}^{J} W_j L_{jY} \\ & \text{subject to} \\ M = & Z_U \left(\prod_{j=1}^{J} L_{jY}^{\eta_{jY}} \right)^{\gamma_Y} \end{aligned}$$

[1] We make this assumption in order to focus on labor demand rather than labor supply, but none of the results would be affected if we were to assume that workers can choose between occupations.

Profits in the upstream sector may be non-zero if there are decreasing returns to scale in the combined labor input ($\gamma_Y < 1$).

Downstream sector The downstream sector consists of a unit measure continuum of identical firms, each of which performs expansion, product differentiation and pricing functions. Expansion departments hire labor from each occupation to manage a measure of product lines, $i \in [0, N]$. Each product line i generates gross profits Π_i, which the expansion department takes as given when deciding on the number of lines to operate. It thus chooses labor L_{jN} and the measure of product lines N to maximize net profits Π_D:

$$\Pi_D := \max_{N, \left\{L_{jN}\right\}_{j=1}^{J}} \int_0^N \Pi_i di - \sum_{j=1}^{J} W_j L_{jN}$$

$$\text{subject to}$$

$$N = Z_D \left(\prod_{j=1}^{J} L_{jN}^{\eta_{jN}} \right)^{\gamma_N}$$

As in the upstream sector, the labor of different occupations are combined with a Cobb-Douglas aggregator, where the occupation weights again satisfy $\eta_{jN} \in [0, 1]$, $\sum_{j=1}^{J} \eta_{jN} = 1$. We allow for an expansion elasticity $\gamma_N \leq 1$ to capture the possibility of decreasing returns to scale in managing product lines from span-of-control or other considerations. Although we will use the language of "expansion" and "product lines" for the N margin, this language should not be interpreted literally. N-type activities represent any means by which downstream firms can replicate gross profits, including developing new products, operating in new geographic markets, marketing to different demographic segments or increasing advertising or sales effort for existing products in existing markets.

The product differentiation department for product line i purchases a quantity m_i of homogenous intermediate goods from the upstream sector, which it costlessly differentiates and sells as final goods y_i to consumers.

The pricing department sets a price p_i in market i, which we describe in terms of a markup $\mu_i \geq 1$ over its marginal cost P_U.[2] Therefore

$$p_i = \mu_i P_U$$

[2]The final goods production function in the downstream sector is therefore $y_i = m_i$, which is why we describe the labor used in the production of intermediate goods as Y-type labor, rather than M-type labor. In Section 3.3, we generalize the model to allow for additional production labor to be used in the downstream sector when differentiating intermediate goods into final goods.

and the gross profits in each product line are given by

$$\Pi_i := p_i y_i - P_U m_i$$

The quantity y_i of differentiated goods sold in market i is determined by the demand conditions in market i and the price p_i. Aggregate output of the downstream sector is

$$Y := \int_0^N y_i di.$$

We will focus only on symmetric equilibria in which $p_i = p \ \forall i$, $m_i = m \ \forall i$, $\mu_i = \mu \ \forall i$ and $y_i = y \ \forall j$, so that $Y = Ny$.

For now, we will treat the markup μ as exogenous and we will remain agnostic about the source of variations in the markup μ, because the theorems that follow about the effects of a change in the markup μ do not depend on a particular micro-foundation. In Section 2.4 we describe various market environments and preferences that are all consistent with this structure. However, readers who prefer to have a concrete example in mind can think of a model of monopolistic competition with a Constant Elasticity of Substitution (CES) aggregator over product lines, analogous to the model of a single firm in Section 1. In that case the markup μ is equal to $\frac{\sigma}{\sigma-1}$, where σ is the elasticity of substitution across varieties σ, and variation in μ arises only from exogenous variation in σ.

2.2 Equilibrium Factor Shares

Market clearing for intermediate goods implies that $mN = M$ and nominal GDP in this economy is given by $pY = pM$. We denote the shares of total income accruing to Y-type labor and N-type labor by

$$S_{LN} := \frac{\sum_{j=1}^{J} W_j L_{jN}}{pY} \text{ and } S_{LY} := \frac{\sum_{j=1}^{J} W_j L_{jY}}{pY},$$

The overall labor share is then given by $S_L = S_{LN} + S_{LY}$. The profit share in the economy is the sum of profit shares in the upstream and downstream sectors, $S_\Pi = S_U + S_D$, where

$$S_U := \frac{\Pi_U}{pY} \text{ and } S_D := \frac{\Pi_D}{pY}.$$

The share of total income accruing to occupation j is denoted by

$$S_j := \frac{W_j L_j}{pY}.$$

Lemma 1. *In a symmetric equilibrium the factor shares satisfy*

$$
\begin{aligned}
S_L &= \frac{1}{\mu}\gamma_Y + \left(1-\frac{1}{\mu}\right)\gamma_N & S_{LY} &= \frac{1}{\mu}\gamma_Y & S_U &= \frac{1}{\mu}\left(1-\gamma_Y\right) \\
S_\Pi &= \frac{1}{\mu}\left(1-\gamma_Y\right) + \left(1-\frac{1}{\mu}\right)\left(1-\gamma_N\right) & S_{LN} &= \left(1-\frac{1}{\mu}\right)\gamma_N & S_D &= \left(1-\frac{1}{\mu}\right)\left(1-\gamma_N\right) \\
S_j &= \frac{1}{\mu}\eta_{jY}\gamma_Y + \left(1-\frac{1}{\mu}\right)\eta_{jN}\gamma_N \ \forall j
\end{aligned}
$$

Proof. See Appendix A.1. □

Lemma 1 shows that the factor shares in this economy are determined by only three parameters: the level of the markup μ and the elasticities of production and expansion with respect to labor, (γ_Y, γ_N). In particular, neither the demand structure that gives rise to the markup μ, nor the relative productivities in the two sectors (Z_Y, Z_N) matter for the income shares. This latter property is a feature of having assumed iso-elastic production functions, which we relax in Section 2.5. Note also that the factor shares in the aggregate economy are identical to the revenue shares for the simple firm problem in Section 1. When $\gamma_N = 0$, the economy collapses to a standard one-sector model with only Y-type labor.

2.3 Effect of the Markup on Labor Income Shares

We use the factor shares in Lemma 1 to answer three questions about the relationship between the markup and the income distribution.

Income redistribution between Y-type and N-type labor How does a change in the markup redistribute income between production and expansionary labor? Theorem 1 shows that whereas the share of income compensating labor used for production activities is negatively exposed to markups, the share of income compensating labor used for expansionary activities is positively exposed to markups. Thus an increase in the markup redistributes income from Y-type to N-type labor.

Theorem 1. *Assume $\gamma_Y, \gamma_N > 0$. An increase (decrease) in the markup μ leads to a decrease (increase) in the income share of Y-type labor and an increase (decrease) in the income share of N-type labor.*

$$\frac{\partial S_{LY}}{\partial \mu} < 0 \ \textit{and} \ \frac{\partial S_{LN}}{\partial \mu} > 0$$

Proof. See Appendix A.2. □

The intuition behind Theorem 1 is that whether the demand for an input rises or falls with an increase in the markup depends on whether the real marginal value of that input to producers (in terms of final goods) is higher or lower when the markup is higher. For the downstream sector, the marginal value of N-type labor is higher when markups are higher (and hence gross profits are higher) because N-type workers allow downstream firms to replicate their existing activities. For the upstream sector, a higher markup translates into a lower value of the intermediate good in units of the final good, which lowers the marginal value of the Y- type labor that is used to produce intermediate goods.

Labor income redistribution between occupations How does a change in the markup redistribute labor income between different occupations? Theorem 2 shows that an increase in the markup redistributes labor income away from occupations that are used relatively more intensively in Y-type activities, and toward occupations that are used relatively more intensively in N-type activities. The relative intensities of an occupation are determined by its production parameters (η_{jY}, η_{jN}). We denote the share of *labor income* accruing to occupation j by $s_j := \frac{S_j}{S_L}$ and refer to these as the occupational labor income shares. Rearranging the factor shares from Lemma 1 results in the following expression relating occupational labor income shares to the markup, from which Theorem 2 then follows

$$s_j = \eta_{jY} + (\eta_{jN} - \eta_{jY}) \left[1 + \frac{\gamma_Y}{\gamma_N} \frac{1}{\mu - 1}\right]^{-1}.$$

Theorem 2. *An increase (decrease) in the markup μ leads to a decrease (increase) in the relative labor income share of occupation j, $s_j := \frac{S_j}{S_L}$, if and only if $\eta_{jY} > \eta_{jN}$*

$$\frac{\partial s_j}{\partial \mu} \lesseqqgtr 0 \text{ if and only if } \eta_{jY} \gtreqqless \eta_{jN}$$

Proof. See Appendix A.3. □

Theorem 2 says that if different occupations have different input shares in Y-type versus N-type activities, then a change in the markup leads to redistribution of labor income across occupations. An increase in the markup redistributes labor income towards occupations that have $\eta_{jN} > \eta_{jY}$, which we refer to as N-intensive occupations. In contrast, we refer to occupations with $\eta_{jY} > \eta_{jN}$ as Y-intensive occupations. Knowledge about which occupations stand to benefit from higher markups requires knowledge of their N-intensity. A corollary of Theorem 2 is that, given knowledge of (γ_Y, γ_N), the co-movement of relative occupational shares s_j and the markup μ is informative about $\eta_{jN} - \eta_{jY}$. In Section 4, we will pursue this strategy to learn about the N-intensity of occupations in the US labor market.

Income redistribution between labor and profits How does a change in the markup redistribute income between labor and profits? Theorem 3 shows that when some workers are engaged in N-type activities ($\gamma_N > 0$), a change in the markup has an ambiguous effect on the overall labor share. In particular, an increase in the markup leads to a fall in the labor share if and only if $\gamma_Y > \gamma_N$. In the special case when $\gamma_N = \gamma_Y$, a change in the markup has no effect on the labor share. And when $\gamma_N > \gamma_Y$, which we will argue below is the empirically relevant case, an increase in the markup leads to an increase in the labor share.

Theorem 3. *An increase (decrease) in the markup μ leads to an increase (decrease) in the overall labor share if and only if the expansion elasticity is bigger than the production elasticity*

$$\frac{\partial S_L}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma_N \gtreqless \gamma_Y.$$

An increase (decrease) in the markup μ leads to an increase (decrease) in the overall profit share if and only if the production elasticity is bigger than the expansion elasticity

$$\frac{\partial S_\Pi}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma_Y \gtreqless \gamma_N.$$

Proof. See Appendix A.4. □

Theorem 3 follows directly from the expression for the overall labor share in Lemma 1, which can be written as $S_L = \gamma_N + (\gamma_Y - \gamma_N)\frac{1}{\mu}$. A corollary is that co-movement of the markup with the labor share is informative about the relative sizes of γ_N versus γ_Y. In Section 3 we will pursue this strategy to learn about the share of total labor income in the US economy that compensates Y-type activities versus N-type activities.

Models that abstract from N-type labor are a special case of our model with an expansion elasticity of zero $(\gamma_N = 0)$. In these economies, the labor share and profit share are given by $(S_L, S_\Pi) = \left(\frac{\gamma_Y}{\mu}, 1 - \frac{\gamma_Y}{\mu}\right)$ so that an increase in the markup μ unambiguously increases the profit share and lowers the labor share. This tight negative relationship between the markup and labor share is strongly engrained in macroeconomic thinking. Empirical work on measuring movements in the markup often equates the markup with the inverse of the labor share (Bils and Klenow, 2004; Nekarda and Ramey, 2019), or estimates the markup as the ratio of the labor (or other variable input) share to the output elasticity of labor (or other variable input) (De Loecker and Warzynski, 2012; De Loecker et al., 2019). Theorem 3 shows that when $\gamma_N > \gamma_Y$, this tight relationship breaks down. In this more general case, the markup μ cannot be inferred from the relationship between the labor share and the production elasticity unless the compensation of Y-type labor is observed separately from N-type labor. Given this point of departure of our model from existing models, in Section 3 we pursue an alternative approach for constructing a proxy for movements in the markup that is valid in the context of our model.

Taking stock Theorems 1, 2 and 3 describe a non-trivial relationship between the markup and the income distribution. The share of labor income paid to some occupations may rise when markups rise, while the income paid to others may fall. Even the share of total income paid to labor may rise or fall in response to an increase in the markup. However these theorems show that the elasticities (γ_Y, γ_N) and the production parameters $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$ fully characterize this pattern of redistribution. Moreover, the theorems suggest an empirical strategy for measuring these parameters and hence learning both about the extent to which labor income compensates Y-type versus N-type activities, and how each occupations' labor income is exposed to movements in the markup. However, before turning to estimation we first discuss alternative interpretations of the source of movements in the markup and describe a series of generalizations of the model.

2.4 Interpretations of Movements in the Markup

We have so far treated the aggregate markup μ as an exogenous wedge between the (competitive) upstream price for intermediate goods produced by Y-type labor, and the downstream price paid by consumers. In this section we describe a number of micro-founded environments in which markup variation arises either as a result of exogenous variation in a structural parameter or for endogenous reasons. Full details of each environment are contained in Appendix B. In each case, Theorems 1, 2 and 3 apply.

We denote the measure of unique varieties consumed by households as Ω and index them by ω. Households have utility defined over an aggregator $C\left(\{c_\omega\}_{\omega\in[0,\Omega]}, \Omega\right)$ that takes consumption of each variety c_ω as input. Recall that N is the measure of product lines operated by downstream firms. We allow for the possibility that $N > \Omega$ to encompass market structures in which more than one downstream firm produces the same variety ω. We focus on symmetric equilibria and denote the measure of downstream product units operating in each variety market as $\mathcal{M} := N/\Omega$. We denote the own-price elasticity of demand for variety ω as

$$\varepsilon\left(\{p_\omega\}_{\omega\in[0,\Omega]},C,\Omega\right):=-\frac{p_\omega}{c_\omega}\frac{\partial c_\omega}{\partial p_\omega}$$

which in a symmetric equilibrium is $\varepsilon(p,C,\Omega)$ and with a homothetic aggregator can be written as $\varepsilon(P,\Omega)$ where $P=\mathcal{P}(p,\Omega)$ is the price index. Below we provide several examples.

Monopolistic Competition Under monopolistic competition, each product line has a monopoly over a single unique variety ($\mathcal{M}=1$, $N=\Omega$) so adding or subtracting product lines N induces one-for-one changes in the measure of goods Ω available for consumption. The markup is given by

$$\mu=\frac{\varepsilon}{\varepsilon-1}.$$

Theorems 1, 2, and 3 apply to movements in the markup due to: (i) exogenous changes in parameters that enter the demand elasticity; or (ii) endogenous changes in variables that enter the demand elasticity (C,Ω,p).[3]

Example 1. With a Constant Elasticity of Substitution (CES) demand system as in Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987), the elasticity takes the form $\varepsilon=\sigma$ where σ is the elasticity of substitution across varieties, so exogenous changes in σ are the only source of markup variation.[4]

Example 2. With a Translog demand system as in Feenstra (2003), Bilbiie et al. (2012) and Maggi and Félix (2019), the elasticity takes the form $\varepsilon=\sigma\Omega+1=\sigma N+1$. Any changes that lead to a decrease in the number of product lines in the economy other than changes to γ_Y or γ_N, such as a fall in Z_D, will induce a rise in markups and the distributional effects in Theorems 1, 2, and 3 will apply. Changes in γ_Y or γ_N also have direct effects on factor shares, but the indirect effects that arise through the resulting change in the markup also satisfy Theorems 1, 2, and 3.

Example 3. With a Linear Demand system as in Melitz and Ottaviano (2008), the elasticity takes the form $\varepsilon=\sigma\frac{\Omega}{C}=\sigma\frac{N}{C}$. Since preferences are not homothetic, the elasticity of demand depends on the level of consumption. Any shock that affects aggregate consumption without directly impacting factor shares (examples of such shocks include shocks to technology (Z_U,Z_D) or shocks to labor supply) leads to a change in the markup and the results of Theorems 1, 2, and 3 hold.

[3] Our theorems also apply in dynamic versions of the model in which the pricing department faces costs of adjusting prices as in Rotemberg (1982) or Calvo (1983) leading to endogenous movements in the markup, as long as the price of new product lines inherits the price of existing lines operated by the same firm.

[4] In a symmetric equilibrium with homogenous goods, the Kimball (1995) demand as used in Klenow and Willis (2016) and Edmond et al. (2018) has a constant markup as in CES.

Cournot Competition Under Cournot competition, each variety ω is produced by a large number of downstream product lines and so $\mathcal{M} = N/\Omega >> 1$. If $\mathcal{M}$ is sufficiently large so that the pricing departments of each product line do not internalize the effects of changes in their own price on the aggregate price index, then the equilibrium markup is

$$\mu = \frac{\varepsilon}{\varepsilon - \frac{1}{\mathcal{M}}}.$$

Consider first the case in which $\mathcal{M}$, the measure of independent downstream product lines selling each variety, is a primitive technological or policy parameter capturing the extent of competition. In this case the creation of new product lines N by retailers generates proportionately more varieties Ω. A change in the markup then arises as a result of exogenous shifts in $\mathcal{M}$, and the distributional and aggregate effects of such changes satisfy Theorems 1, 2, and 3.

Alternatively, one could consider the measure of unique varieties Ω as a primitive. In this case, the creation of new product lines by downstream firms generates a proportionate increase in $\mathcal{M}$, the measure of sales units competing in each market, and hence leads to a fall in the markup. Theorems 1, 2, and 3 apply to this change in the markup. An example of such a shock would be a shock to the relative productivities in the two sectors $\frac{Z_U}{Z_D}$.

Oligopoly Oligopoly refers to the case in which each variety ω is produced by $\mathcal{M} > 1$ downstream product lines, but $\mathcal{M}$ is sufficiently small that retailers take strategic considerations into account when setting prices. As in the previous case we can treat either the number of independent downstream product lines selling each variety $\mathcal{M}$, or the total measure of unique varieties Ω, as the primitive. We focus on the nested CES case as in Atkeson and Burstein (2008), Jaimovich and Floetotto (2008) and Mongey (2019), in which the elasticity of substitution across the same variety sold by different firms is given by $\vartheta > \sigma$. The previously considered case in which the same varieties sold by different retailers are perfect substitutes corresponds to $\vartheta \to \infty$.

Example 4. Under Bertrand competition the markup is given by

$$\mu = \frac{\vartheta + \frac{\sigma - \vartheta}{\mathcal{M}}}{\vartheta - 1 + \frac{\sigma - \vartheta}{\mathcal{M}}}$$

which gives $\mu = 1$ when $\vartheta \to \infty$.

Example 5. Under Cournot competition the markup is given by

$$\mu = \frac{\sigma\vartheta}{\sigma\left(\vartheta - 1\right) + \frac{\sigma - \vartheta}{\mathcal{M}}}$$

which gives $\mu = \frac{\sigma}{\sigma - \frac{1}{M}}$ when $\eta \to \infty$.

In either case a change in the markup arising from a change in either demand elasticity, or from a change in the degree of concentration M, induces the distributional effects implied by Theorems 1, 2, and 3.

Limit Pricing As in Barro and Tenreyro (2006), we assume that there exists an alternative technology for downstream firms to operate a product line that does not require hiring any N-type labor. Instead the downstream firm incurs additional input costs so that its effective marginal cost of undifferentiated goods is κP_U, with $\kappa > 1$. This captures, for example, the costs of licensing an existing product from abroad to sell in a new market, or the costs of trying to compete in a product market without setting up the necessary sales infrastructure or overhead. If $\kappa < \mu$ in any of the aforementioned market structures, then the equilibrim markup is κ and any change in κ will generate the redistributive effects described in Theorems 1, 2, and 3.

2.5 Generalizations of Production Structure

Our assumed production structure contains several special features that are not strictly necessary for a change in the markup to have the redistributive effects described in Theorems 1, 2, and 3. Relaxing these features is useful for understanding the economic forces at work. For simplicity we describe these generalizations in the context of monopolistic competition with $N = \Omega$.

Variety-specific DRS in production Our baseline model features a homogenous intermediate good that is then differentiated. This implies that DRS in the use of Y-type labor operates at the economy-wide level. In the absence of love-of-variety in preferences, the production of new product lines is a socially wasteful activity and a planner would choose to set $N \to 0$. Some readers may find this feature of our environment unappealing. However, this assumption is not important for Lemma 1 or for Theorems 1, 2, or 3. In Appendix C.1 we describe an alternative version of the model, in which each variety is produced with a separate DRS production function in the upstream sector. We show that in this alternative model, the factor shares are identical to those in the baseline model. With this alternative formulation, there is indeed a social benefit to introducing new product lines, and a planner would choose a value of $N > 0$ as in Bilbiie et al. (2016). But the distributional effects of a change in the markup are not affected.

Integrated wholesale and retail sectors as single firms The model described above differs from the one shown in the introduction in that it assumes that decisions about how much Y-type labor to hire are made independently of decisions about how much N-type labor to hire. When a downstream firm is deciding about whether to expand its number of product lines, it does not internalize the effect that this will have on the marginal cost of production. We separated decisions about production and expansion to facilitate using data on Producer Price Indices as a proxy for markups in Section 3. In Appendix C.2, we analyze a general equilibrium version of the model in Section 1, in which a continuum of firms each decide jointly on production and expansion. In this model, the expressions for the factor shares in Lemma 1 and Theorems 1, 2 and 3 are the same as in the baseline model.

Capital and other factors of production So far we have considered a production structure in which labor is the only factor used in production and expansion. In Appendix C.3, we describe a more general version of the model that incorporates other factors such as capital and materials. We show that if the production and expansion functions are weakly separable between labor and other factors of production and expansion, then implications for labor shares in Theorems 1, 2 and 3 are unaffected. Specifically, we assume the production and expansion functions take the form

$$
\begin{aligned}
M &= Z_U L_Y^{\gamma_Y} f_Y\left(X_Y\right) \\
N &= Z_D L_N^{\gamma_N} f_N\left(X_N\right)
\end{aligned}
$$

where X_Y and X_N are vectors of inputs used for production and expansion respectively, and f_Y and f_N are arbitrary functions. In this case, the expressions for the labor share are the same as in Lemma 1. Profit shares, however, will depend on the functions f_Y and f_N.

Other Generalizations In Section 3.3, we describe and estimate an extension of the model that allows additional Y-type labor to be used in the differentiation of goods in the downstream sector. In Section 4.3, we describe and estimate an extension of the model that incorporates different industries and allows for industry heterogeneity in the importance of N-type versus Y-type labor. We have also explored the effects of various other generalizations of the model on Theorems 1, 2 and 3. These include allowing for Constant Elasticity of Substitution production functions (C.4), entry in the upstream sector (C.5), and markups in both the labor market and the market for upstream goods (C.6). Details of these generalizations, along with the modifications to the conditions of Theorems that they require, are contained in Appendix C.

3 Estimation of Aggregate Parameters

We estimate the model parameters in two steps. We start in this section by estimating the production and expansion elasticities (γ_Y, γ_N) using time series data on the overall labor share S_L and a proxy for the markup μ that we will describe below. Knowledge of these parameters, together with the average markup, is then sufficient to infer the aggregate fraction of US labor income that compensates N-type activities. Then, in Section 4, we estimate the parameters that govern the N-type and Y-type content of each occupation, $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$, given estimates of (γ_Y, γ_N) and data on occupational labor income shares.

The philosophy underlying our estimation strategy in this and the following section is premised on the notion that the scope of N-type labor encompasses too broad a range of activities, and the distinction between Y-type and N-type activities too abstract, to be able to directly observe payments to each type of labor separately in available data. Attempting to do so might involve using data on workplace tasks from O*NET or measuring the contribution of different occupations to specific corporate activities (such as R&D, product design, overhead, sales and marketing). We choose not to take this route because we do not presume to know ex-ante the mapping from specific tasks to N-type versus Y-type activities. Instead, we go the opposite route, inferring the aggregate share of N-type labor income, as well as the N-type and Y-type content of different occupations, by exploiting the model predictions underlying Theorems 1, 2 and 3. According to the model, N-type activities are those whose share of total labor compensation increases in response to a markup-induced increase in the overall labor share, whereas Y-type activities are those whose share of total labor compensation falls in response to a markup-induced increase in the overall labor share.

The variation we exploit is aggregate time-series fluctuations at business cycle frequencies. We focus on business cycle frequencies because all of the key variables used in estimation (aggregate labor share, average aggregate markup, occupational labor income shares) exhibit strong secular trends which have been the focus of extensive existing literatures, and because cyclical movement in markups lie at the heart of a large class of modern business cycle models. Our model is purposely as simple as possible and features no endogenous dynamics. We thus treat it as a repeated static economy for the purpose of estimation.

3.1 Identification and Estimation of Overall Shares (γ_Y, γ_N)

As discussed in Section 2.2, the factor share equations can be re-arranged to give the following expression relating the markup to the labor share,

$$S_L = \gamma_N + (\gamma_Y - \gamma_N)\frac{1}{\mu}. \tag{1}$$

This equation underscores our strategy for identifying (γ_Y, γ_N). Given variation in the markup μ that arises from any of the sources described in Section 2.4 that do not involve a change in (γ_N, γ_Y), equation (1) implies that (γ_N, γ_Y) are identified by the average levels of the markup and the labor share, and the co-movement of the labor share with the markup. Intuitively, Lemma 1 showed that the factor shares are determined by $(\gamma_N, \gamma_Y, \mu)$ and thus three moments are required. The levels of the labor share and the markup provide two of these. The third moment exploits the insight of Theorem 3: the sign and strength of co-movement between the labor share and the markup identifies the gap between γ_N and γ_Y.

Our estimating equation is the empirical counterpart to (1) is

$$\begin{aligned} S_{L,t} &= \gamma_N + (\gamma_Y - \gamma_N)\frac{1}{\mu_t} + \epsilon_{L,t} \\ \frac{1}{\mu_t} &= \frac{1}{\mu} + \epsilon_{\mu,t}, \end{aligned} \tag{2}$$

where $S_{L,t}$ and μ_t are de-trended series for the aggregate labor share and markup.[5] According to this equation, cyclical variation in the measured labor share could arise because of variation in the markup due to one of the forces described in Section 2.4 (captured by $\epsilon_{\mu,t}$) or because of measurement error in the labor share or cyclical variation in other factors outside the model (captured by $\epsilon_{L,t}$). The assumption required for identification of (γ_N, γ_Y) is that variation in the inverse markup $\epsilon_{\mu,t}$ is independent of these other sources of cyclical variation in the labor share $\epsilon_{L,t}$

$$E[\epsilon_{L,t}] = 0 \;\forall t \tag{3}$$

$$E[\epsilon_{L,\tau} | \epsilon_{\mu,t}] = 0 \;\forall (t,\tau). \tag{4}$$

These moment conditions form the basis of our estimation strategy. Note that the im-

[5] A natural question is how to formally incorporate trends into the model and derive equation (7) in terms of de-trended data. In Appendix F we show that under the assumption that γ_Y and γ_N grow at a common deterministic trend, the model implies an equation for the de-trended markup and aggregate labor share as in equation (7)

plicit assumption underlying this strategy is that the production and expansion elasticities (γ_Y, γ_N) do not vary at business cycle frequencies. Assuming that technological parameters such as these are constant over the business cycle has a long tradition in economics - it is imposed, either explicitly or implicitly, in almost all existing empirical exercises that use Real Business Cycle and New Keynesian models.

3.2 Labor Share and Markup Data

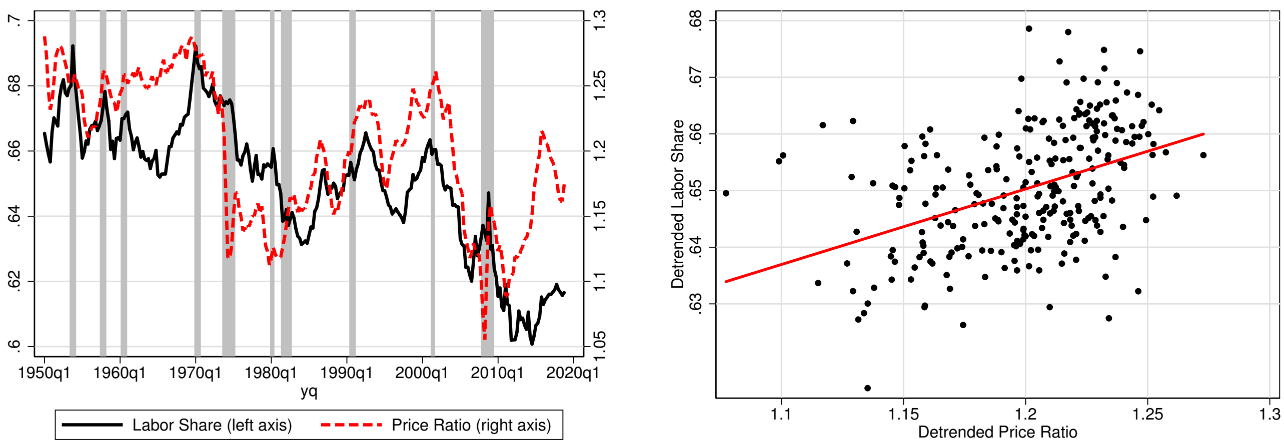
To implement this estimation strategy, we require de-trended data on the labor share and the markup.

Labor share data We construct our baseline measure of the labor share using quarterly data from the National Economic Statistics produced by the Bureau of Economic Analysis, following the procedures in Gomme and Rupert (2004) to adjust for ambiguous components of income. We use data for 1950:Q1 to 2018:Q4 and de-trend using the method in Hamilton (2018). All of our estimates are robust to using alternative measures of the labor share and alternative methods for de-trending. Appendix E.1 contains full details of the construction of the series and Appendix E.2 reports estimates using alternative data series for the labor share.

The raw series for the labor share is displayed in Figure 1a (black solid line, left axis). The mean of this series over the sample is 65.1%. The series displays the well-documented downward trend in the labor share, from an average of 67.0% pre-1960 to 61.2% post-2010. The labor share is also counter-cyclical, which can be seen in Figure 1a, by noting that the NBER recessions (shaded grey areas) typically correspond to local maxima of the series. The correlation of the de-trended labor share series with de-trended log per-capita real output is -0.13. This counter-cyclicality of the labor share is consistent with a large body of evidence.

Markup data Estimating markups and their co-movement with aggregate economic activity has a long and controversial history in economics. We refer the reader to the recent paper by Nekarda and Ramey (2019) for an excellent discussion of the relevant literature and different approaches to estimating the dynamics of the markup at business cycle frequencies.[6] Unfortunately, none of the approaches used in this literature are appropriate in

[6]Nekarda and Ramey (2019) classify four approaches that have been used to estimate a time-series for the markup: (i) using direct data on price and average variable costs; (ii) generalizations of the Solow residual as in Hall (1986); (iii) generalized production functions with quasi-fixed factors; (iv) using factor share equations, adjusted for fixed costs.



(a) Raw time series

(b) De-trended data

Figure 1: Panel (a): Raw time series for labor share and markup. Labor share (left axis) is computed from BEA data following Gomme and Rupert (2004). Price ratio (right axis) is the ratio of PPI series WPSFD49207 to series WPSID61. Shaded areas are NBER recessions. Panel (b): De-trended labor share and de-trended price ratio. Labor share and inverse price ratio are de-trended using the Hamilton filter.

the context of our model. Most existing approaches require the researcher to specify which are the variable factors that are used in the production of goods, as opposed to factors that contribute to generating revenue for firms in ways other than variable production. In our model this would require distinguishing between payments to N-type versus Y-type labor as a pre-requisite to estimating the markup.[7] Rather than learning about the markup by assuming that Y-type labor is observed, we instead seek to learn about the split between N-type versus Y-type from movements in the markup. We thus cannot adopt one of these existing methods and use it as an input to our estimation procedure.

Since we require a markup series that has been constructed without assuming away the existence of N-type inputs, we instead construct a proxy for the markup by comparing the prices of goods at different stages of the production process, similarly to Barro and Tenreyro (2006). Let ϱ_t denote the ratio of p_t (the price of differentiated final goods that are sold to consumers) to $P_{U,t}$ (the price of undifferentiated goods produced from raw inputs). In our baseline model, the price ratio ϱ_t is identically equal to the markup μ_t

[7]See Bond et al. (2021) for an explanation of why the methods used in De Loecker and Warzynski (2012) and De Loecker et al. (2019) cannot be used in the context of our model, unless one could separately observe payments to N-type and and Y-type labor. Using an input other than labor as the variable factor of production leads to similar difficulties in the context of our model, unless the payments to those inputs that compensate production activities can be observed separately from those that compensate expansionary activities. This excludes, for example, the use of intermediate inputs as in Bils et al. (2018).

$$\varrho_t := \frac{p_t}{P_{U,t}} = \mu_t.$$

We therefore construct an empirical counterpart to ϱ_t which we use as a proxy for the markup μ_t in estimation. We use indexes produced by the Bureau of Labor Statistics (BLS) Producer Price Index (PPI) program that closely match the model definitions of p_t and $P_{U,t}$ and are available since 1947. For the numerator, we use the series for "Finished demand" (WPSFD49207), which measures the price changes of goods for (i) personal consumption and (ii) capital investment. For the denominator, we use the series for "Processed goods for intermediate demand" (WPSID61), which measures the price changes of (i) partially processed goods that have to undergo further processing before they can be sold to the public and (ii) supplies consumed by businesses.

Because our proxy for the markup is a price ratio that is constructed by comparing two indexes, it allows us to measure changes in the markup over time but does not provide an estimate of the *level* of the markup, which is also required for our estimation procedure. For our baseline estimates, we assume that the average markup over the post-war period is 1.2, and we show how our estimates are affected by assuming a value between 1.05 and 1.35. It turns out that the level of the markup has very little effect on the estimates of (γ_N, γ_Y), since these are identified by the co-movement of the markup with the labor share, and the level of the labor share. It also has a negligible effect on our estimates of the occupation-level parameters in Section (4) that determine the relative N-intensity of different occupations. However, the level of the markup does matter for the implied estimate of the fraction of total labor payments that compensate N-type labor, $\frac{S_{LN}}{S_L}$.

The raw price ratio series is displayed in Figure 1a (red dashed line, right axis), rescaled to have a mean of 1.2. The series displays a slight downward trend. The series is also counter-cyclical and co-moves positively with the labor share at business cycle frequencies. The correlation of the de-trended price ratio with de-trended log per-capita real output is -0.28, and the correlation with the de-trended labor share is 0.28. This positive co-movement between our proxy for the markup and the labor share is the key feature of the data suggesting that $\gamma_N > \gamma_Y$ and that the share of N-type labor is positive.[8]

[8] The counter-cyclicality of our markup series is not at odds with the conclusions in Nekarda and Ramey (2019) that the markup based on labor compensation is pro-cyclical, since this is simply the inverse of the labor share which is also pro-cyclical in our data.

	(1) Baseline	(2) Low Markup	(3) High Markup	(4) Short Sample (1984-2018)	(5) Annual Averages	(6) Five-Year Averages	(7) Ten-Year Averages
γ_Y	0.620	0.643	0.597	0.613	0.624	0.612	0.619
	(0.005)	(0.001)	(0.009)	(0.006)	(0.009)	(0.011)	(0.014)
γ_N	0.804	0.804	0.804	0.832	0.783	0.846	0.806
	(0.025)	(0.025)	(0.025)	(0.030)	(0.043)	(0.055)	(0.065)
Implied $\frac{S_{LN}}{S_L}$	21%	6%	32%	22%	20%	22%	21%
P-value $\gamma_Y = \gamma_N$	0.00	0.00	0.00	0.00	0.00	0.02	0.06
Mean markup μ	1.20	1.05	1.35	1.20	1.20	1.20	1.20

Table 1: First step estimation results
This table shows the estimates for (γ_N, γ_Y) in Equation (2). In columns (2) to (3) we vary the assumed mean markup used in construction of $\frac{1}{\mu_t}$. In column (4) we restrict the sample to 1984:Q1-2018:Q4. In columns (5) to (7) we use non-overlapping one-, five- and ten-year averages of variables $S_{L,t}$ and $\frac{1}{\mu_t}$. Reported standard errors are robust.

3.3 Estimates of Production and Expansion Elasticities (γ_Y, γ_N)

Baseline estimates We estimate equation (2) by OLS, imposing the constraints that $\gamma_N, \gamma_Y \leq 1$.[9] Our baseline estimates are shown in the first column of Table 1. We estimate a value for γ_N of 0.80 and a value for γ_Y of 0.62. Figure 1b displays a scatter plot corresponding to this regression, which illustrates the positive co-movement between the de-trended labor share and de-trended price ratio. The higher estimate for γ_N than γ_Y is a direct consequence of this positive co-movement. Given our assumptions of a mean markup of 1.2, these estimates imply that roughly one-fifth of labor income compensates expansion activities as opposed to production activities.

The second and third columns of Table 1 show that the estimate of the share of labor income compensating N-type activities is sensitive to the assumption about the mean markup. If one believes that the mean level of the markup is lower (higher) than 1.2, then one would obtain lower (higher) estimates of this share. However, the estimates of the elasticities γ_Y, γ_N are not sensitive to the assumed level of the markup, and it is these

[9] These constraints do not bind at our baseline estimates but do bind in some of the estimates by industry in Section 4.3.

elasticities that are required as input into the second step occupation-level estimation in Section 4. In the fourth column we report estimates from a restricted sample starting in 1984, which corresponds to the time period for which we have occupation level data that will be used in the second step. The estimates from this more recent sub-period are very close to the baseline. In the remaining columns we report estimates using non-overlapping one-year, five-year and ten-year averages of the quarterly time series. The estimates are not sensitive to using data that is averaged over longer horizons. This helps to alleviate concerns that differential adjustment costs between labor used in production versus expansion might alter the interpretation of our results, because adjustments costs are arguably less relevant over these longer horizons.

Another concern may be that because we use data at business cycle frequencies, the labor share movements that drive the production and expansion elasticity estimates reflect unmodelled general business cycle forces and not movements in the markup per se. To alleviate such concerns, Table 8 in Appendix E.2 reports estimates of equation (2) controlling for GDP, lags of GDP, and interactions of GDP and lagged GDP with our markup proxy. The estimates are barely changed by the inclusion of these controls, suggesting that the variation in the price index ratio our estimates exploit is separate to quarterly variation in GDP. Tables 6 and 7 in Appendix E.2 show that none of our estimates are sensitive to using alternative series for the labor share or alternative assumptions about how we de-trend the data.

Downstream processing One valid concern with our use of the price ratio ϱ as a proxy for the markup μ is that the equality of the markup and the price ratio in our model relies on the assumption that Y-type labor is used only in the production of intermediate goods in the upstream sector and is not required for the differentiation of intermediate goods into final goods. If Y-type labor is also used for further processing in the downstream sector, then the value added of this additional labor input would contribute to the price of downstream goods. In this case, marginal costs in the downstream sector would include labor costs alongside the cost of intermediate goods. The price ratio ϱ would therefore confound the markup with the wages paid to downstream production workers. We think it is important to take this possibility into account and ensure that it is not driving our findings. To do so, we consider an extension of the model that allows for Y-type labor to be used also for product differentiation in the downstream sector.

We assume that the production function for final goods uses both intermediate goods and Y-type labor of each occupation as inputs, according to the Cobb-Douglas aggregator

$$y_i = m_i^{1-\gamma_D} \left(\prod_{j=1}^{J} \ell_{jiYD}^{\eta_{jY}} \right)^{\gamma_D} .$$

The downstream production elasticity γ_D governs the importance of production labor in the downstream sector. The baseline model is a special case of this more general model with $\gamma_D = 0$. Full details of this more general model are contained in Appendix C.7. There we also show that when $\gamma_D > 0$, Theorems 1, 2, and 3 all continue to hold, but with the composite production elasticity $\gamma_D + (1 - \gamma_D)\,\gamma_Y$ taking the place of γ_Y. Although the price ratio ϱ is no longer equal to the markup μ, the two are related according to the formula

$$\frac{1}{\mu} \propto \left(\frac{1}{\varrho} \right)^{1-\gamma_D} \left(\frac{W_Y}{p} \right)^{\gamma_D} \tag{5}$$

where $W_Y := \prod_{j=1}^{J} \left(\frac{W_j}{\eta_{jY}} \right)^{\eta_{jY}}$ is the wage index for Y-type labor. Therefore to use the price ratio as a proxy for the markup, we require data on the real wages of production labor in the downstream sector and a value for the production labor elasticity in the downstream sector γ_D. For real wages, we use average hourly compensation in the non-farm business sector from the BLS deflated by the PPI series for "Finished demand" (WPSFD49207), but our results are robust to using average wages of production and nonsupervisory employees.[10] Since we do not have a way to estimate the downstream production elasticity γ_D, we report results for values various assumed values of γ_D , as well as for a specification where we estimate $(\gamma_Y, \gamma_N, \gamma_D)$ under the restriction that the production elasticities are the same in the upstream and downstream sectors ($\gamma_Y = \gamma_D$). Figure 4 in Appendix E.2 shows the time series for the markup implied by the price ratio for each of these values. Table 2 contains estimation results for each value for γ_D. Raising γ_D above zero lowers the estimate of γ_Y from its baseline estimate of 0.62, and when γ_D is restricted to be equal to γ_Y, the two are equal to 0.38. However, for all values of γ_D, the composite elasticity $\gamma_D + (1 - \gamma_D)\,\gamma_Y$ that determines the overall share of Y-type labor is barely affected and hence neither are our conclusions about the overall share of labor income compensating N-type labor.

[10] Average Hourly Compensation in the Non-Farm Business Sector, from the BLS (PRS85006103) is provided as an index (equal to 100 in 2012). Average Hourly Earnings of Production and Nonsupervisory Employees, Total Private, from the BLS (CES0500000008) is expressed in dollars per hour. Production and related employees include working supervisors and all nonsupervisory employees. Nonsupervisory employees include those individuals in private, service-providing industries who are not above the working-supervisor level. We use WPSFD49207 as it corresponds to p in equation (5). We construct the measure of markup by first de-trending the logarithm of the right hand side of equation (5) and then by adding a constant to the logarithm of the remaining cyclical component to ensure that, on average, the markup is equal to a chosen value.

	(1) Baseline	(2) $\gamma_D = 0.10$	(3) $\gamma_D = 0.25$	(4) $\gamma_D = 0.33$	(5) (GMM) $\gamma_D = \gamma_Y$
γ_Y	0.620	0.577	0.495	0.442	0.376
	(0.005)	(0.006)	(0.009)	(0.012)	(0.007)
γ_N	0.804	0.809	0.772	0.744	0.844
	(0.025)	(0.027)	(0.033)	(0.041)	(0.038)
Implied $\frac{S_{LN}}{S_L}$	21%	21%	20%	19%	22%
Implied $(1-\gamma_D)\,\gamma_Y + \gamma_D = \gamma_N$	0.620	0.619	0.621	0.622	0.610
P-value $(1-\gamma_D)\,\gamma_Y + \gamma_D = \gamma_N$	0.00	0.00	0.00	0.00	0.00
Mean markup, μ	1.20	1.20	1.20	1.20	1.20

Table 2: First step estimation results, $\gamma_D > 0$

This table shows the estimates for (γ_N, γ_Y) in Equation (2) for measures of the markup given by equation (5). In column (1) we show the estimates from the baseline specification with no downstream processing ($\gamma_D = 0$). In columns (2) - (5) we vary γ_D used in construction of the measure of the markup. In column (5) we show optimally-weighted GMM estimates for $(\gamma_N, \gamma_Y, \gamma_D)$ with restriction $\gamma_D = \gamma_Y$. Reported standard errors are robust.

Taking Stock What should we take away from these estimates? Conventional macroeconomic models implicitly assume that $\gamma_N = 0$ and therefore that all payments to labor compensate Y-type activities. But our estimates imply that γ_N is substantially larger than zero. Indeed, our estimates imply that the labor expansion elasticity is larger even than the labor production elasticity, γ_Y (or its relevant composite $\gamma_D + (1 - \gamma_D)\,\gamma_Y$). These estimates follow directly from the positive co-movement between the overall labor share and our proxies for the markup at business cycle frequencies. It follow that unless one believes that the aggregate markup is very small, one must conclude that a non-trivial fraction of the US labor force is engaged in N-type activities. This means that some parts of the labor force stand to gain, while others stand to lose, from a change in markups, whether they arise from cyclical shifts in aggregate demand, or from structural changes induced by changes in the competitiveness of product markets. In the next section, we use an occupational lens to shed light on which parts of the labor force are which.

4 Estimation of Occupation-Specific Parameters

4.1 Identification and Estimation of Occupational Factor Shares $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$

In Section 2.3 we derived an expression relating the occupational labor income shares s_j to the markup. Combining that expression with equation (1) that relates the overall labor share S_L to the markup yields the following relationship between occupational labor income shares and the overall labor share,

$$s_j = \eta_{jY} + (\eta_{jN} - \eta_{jY}) \left(1 - \frac{\gamma_Y}{\gamma_N}\right)^{-1} \left(1 - \gamma_Y \frac{1}{S_L}\right) \ \forall j. \tag{6}$$

Equation (6) underlies our approach to estimation of $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$. It shows that for given values of the production and expansion elasticities (γ_N, γ_Y), we can recover (η_{jN}, η_{jY}) from the average occupational labor income shares and the co-movement of occupational labor income shares with any variation in the labor share S_L that is independent of variation in the occupational factor share parameters (η_{jN}, η_{jY}). As in the first step in Section 3, we work with de-trended data. Cyclical variation in measured occupational labor income shares s_j could arise from measurement error in occupational income shares, shocks to (η_{jN}, η_{jY}), or variation in the overall labor share S_L arising from any of the sources described in the previous section.

Our estimating equation is the empirical counterpart to equation (6),

$$\begin{aligned}
s_{j,t} &= \eta_{jYt} + (\eta_{jN,t} - \eta_{jY,t}) \left(1 - \frac{\gamma_Y}{\gamma_N}\right)^{-1} \left(1 - \gamma_Y \frac{1}{S_{L,t}}\right) + \epsilon_{s_j,t} \ \forall j \qquad (7) \\
\eta_{jY,t} &= \eta_{jY} + \epsilon_{jY,t} \\
\eta_{jN,t} &= \eta_{jN} + \epsilon_{jN,t},
\end{aligned}$$

where $\epsilon_{Y,t} := (\epsilon_{1Y,t} \ldots, \eta_{JY,t})'$, $\epsilon_{N,t} := (\epsilon_{1N,t} \ldots, \eta_{JN,t})'$ and $\epsilon_{s,t} := (\epsilon_{s_1,t} \ldots, \eta_{s_J,t})'$ are mutually independent $J \times 1$ random vectors that are IID over time and each sum to zero. We define $\epsilon_{j,t} := \left(\epsilon_{jY,t}, \epsilon_{jN,t}, \epsilon_{s_j,t}\right)$' and assume that $E\left[\epsilon_{j,t}\right] = 0 \ \forall t, \ \forall j$.[11] We now describe three different sets of moment conditions that can be used as the basis for estimation. Each differs in the type of variation in the labor share that it uses.

De-trended Markup Proxy From equations (1) and (6), we see that movements in the markup affect occupational labor shares only through their effect on the overall labor share. Thus a valid source of variation in the labor share that can be used for identification is markup-induced variation. To exploit such variation we can use our proxies for the markup based on the ratio of price indices as an instrument for the de-trended labor share in equation (7). This identification strategy thus imposes the moment condition

$$E\left[\epsilon_{j,\tau} \middle| \epsilon_{\mu,t}\right] = 0 \ \forall \left(t, \tau\right), \ \forall j. \qquad (8)$$

We choose this specification as our baseline because it uses the same variation to estimate the occupational factor share parameters $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$ as we used to estimate the overall factor share parameters (γ_Y, γ_N) in Section 3.

De-trended Labor Share A simple alternative approach is to assume that the de-trended labor share $S_{L,t}$ is itself orthogonal to shocks to the occupational factor share parameters and measurement error in the occupational income shares $s_{j,t}$. This imposes the moment condition

$$E\left[\epsilon_{j,\tau} \middle| S_{L,t}\right] = 0 \ \forall \left(t, \tau\right), \ \forall j \qquad (9)$$

Because we work with de-trended data, equation (9) says that any movements in occupational labor shares $s_{j,t}$ that are correlated with movements in the overall labor share must be at lower frequencies than those removed by our de-trending procedure. Recall from equation

[11] An example that satisfies these assumptions is that $\epsilon_{Y,t}$, $\epsilon_{N,t}$ and $\epsilon_{s,t}$ are each drawn from translated Dirichlet distributions.

(6) that low-frequency variation in the labor share can arise from trends in the production parameters or the markup. The assumption requires that business cycle variation in the labor share does not arise from sources that directly affect the occupational income shares, other than through the channel in equation (6). Through the lens of the model, this means that business cycle variation in the labor share must come from variation in the markup μ, rather than from the elasticity parameters (γ_Y, γ_N). As discussed above, assuming that technological parameters are fixed at business cycle frequencies is a common assumption.

We must also assume that the shocks to individual occupation shares $(\epsilon_{Y,t}, \epsilon_{N,t})$ are independent of the de-trended overall labor share $S_{L,t}$. This is a relatively weak assumption because the random vectors $(\epsilon_{Y,t}, \epsilon_{N,t})$ each sum to zero - so failure of this assumption would require a re-shuffling of occupations at exactly the same time as a shock to the labor share, without any change in the markup. Finally, we need to assume that measurement error in the occupational labor income shares is independent of measurement error in the overall labor share. This assumption is likely to be satisfied because we use different data sources to measure $s_{j,t}$ and $S_{L,t}$, rather than constructing a measure of $S_{L,t}$ by summing over $S_{j,t}$.

Monetary Policy Shocks Given estimates of (γ_Y, γ_N), it is also possible to estimate the occupational factor shares without data on the markup. Assume that a variable Z_t is available, which is related to the inverse markup by

$$\frac{1}{\mu_t} = \frac{1}{\mu} + \zeta Z_t + \epsilon_{\mu,t}, \tag{10}$$

where $\zeta \neq 0$, so that there is a valid first-stage, and with $E[Z_t] = 0$. Then if the moment condition

$$E[\epsilon_{j,\tau} | Z_t] = 0 \; \forall (t, \tau), \; \forall j \tag{11}$$

holds, we can estimate (η_{jN}, η_{jY}) by using Z_t as instrument for the labor share in equation (7). This assumption requires that the instrument Z_t only affects the occupational labor income shares through its effect on the overall labor share, which in our framework can only occur if the instrument causes a change in the markup.

Two types of variables that are likely to satisfy these assumptions are monetary and fiscal policy shocks. In general equilibrium models with sticky prices, such as New Keynesian DSGE models, contractionary monetary and fiscal policy shocks (as well as other contractionary demand shocks) generate a rise in the markup (so $\zeta \neq 0$), and do not affect the labor share except through their effect on the markup.[12]

[12]This is true in a broad class of New Keynesian models. See for example Christiano et al. (2005), Smets

Monetary policy shocks in particular are a good candidate instrument for the labor share. Cantore et al. (2021) undertake a comprehensive empirical investigation of the dynamic effects of a monetary policy shock on the labor share. Using various different strategies for identifying monetary policy shocks, they document robust evidence that a contractionary monetary shock leads to an increase in the labor share, with a peak response after 1-2 years. They also show that standard New Keynesian models (with $\gamma_N = 0$) cannot reproduce those dynamics, exactly because a contractionary monetary shock is associated with an increase in the markup, which is incompatible with a fall in the labor share in standard models. However, in a New Keynesian model with our production structure, and the parameters estimated in Section 3, a contractionary monetary shock does lead to rise in the labor share. Thus, given $\gamma_N > \gamma_Y$, monetary policy shocks can be used as an instrument for the labor share.

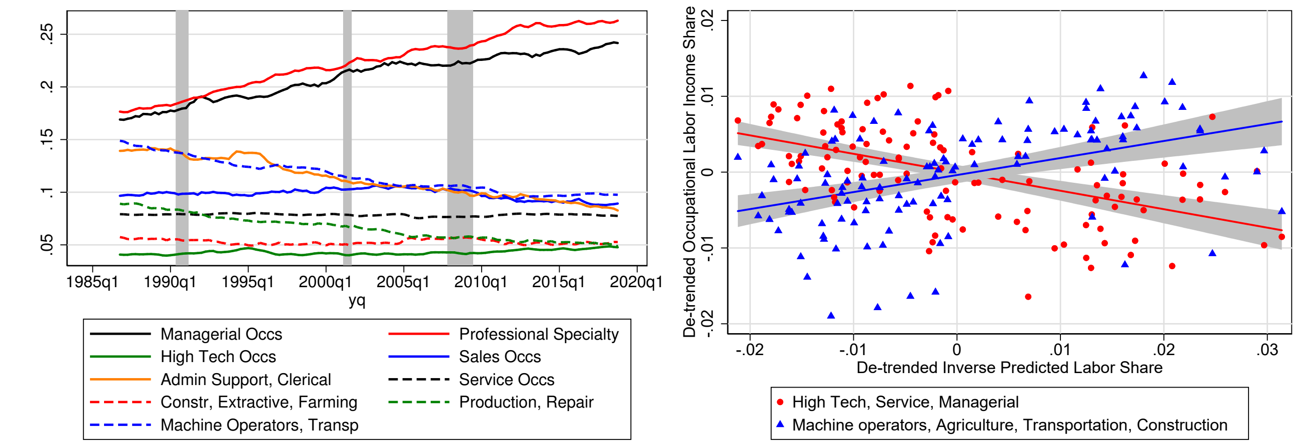
Based on the dynamic responses in Cantore et al. (2021) we use two monetary shock series produced with two different strategies: (i) Romer and Romer (2004), extended by Coibion et al. (2017), and (ii) Gertler and Karadi (2015). For each series we also include lags at horizons of 1 to 4 quarters, for a total of 10 instruments, and estimate the parameters using optimally-weighted GMM. The instrument set has a strong first stage and we fail to reject the test of over-identification restrictions.

4.2 Data on Occupational Income Shares

We use data from the monthly Outgoing Rotation Groups of the Current Population Survey (CPS-ORG) to construct quarterly series for occupational income shares s_j. We restrict attention to employed individuals aged 16 and over, and we measure labor income with the IPUMS variable 'earnweek', which records gross weekly earnings on the respondent's main job. We compute labor income for individuals in each of 9 broad occupation categories, which we construct from the 389 OCC1990 occupation codes. We then aggregate monthly earnings in each occupation to the quarterly level and compute the occupation shares s_j in each quarter from 1984 to 2019. We de-seasonalize the quarterly series and then detrend using a Hamilton filter. The trend components for each of the 9 occupation groups are displayed in Figure 2a. Managerial and professional specialty occupations display the strongest growth in income shares in our sample, while machine operators, transportation, administrative and clerical occupations show the steepest decline.

Figure 2b shows a scatter plot of the cyclical components of the occupational income shares that illustrates our identification strategy. To construct this figure we split the

and Wouters (2007) and Galí et al. (2015).



(a) Trend components of occupational share s_j.

(b) Cyclical components of occupational shares s_j.

Figure 2: Panel (a) Trend components of occupational labor income shares. Trends extracted using a Hamilton filter. Panel (b): Horizontal axis is predicted value of the inverse of the overall labor share from OLS regression of the inverse labor share on markup. Vertical axis is cyclical component of occupational labor income shares. Occupations grouped into two broad categories based on estimates of N-intensity in Table 3. Shaded areas are 95% confidence intervals (robust standard errors are used).

occupations into two broad groupings, based on our baseline estimates of the occupational factor share parameters. For each of the two groups, we plot the de-trended labor income share of those occupations against the predicted value of the de-trended overall inverse labor share, from an OLS regression of the inverse labor share on the markup. Thus the fitted line reflects the IV estimate of equation (7) using the markup as an instrument, and the slope of the relationship reveals the sign of $\eta_{jN} - \eta_{jY}$.

The group represented by the red circles in Figure 2b consists of the most N-intensive occupations and comprises approximately 32% of total labor income (high-tech, services and managerial). The relatively large N-component for these occupations is revealed by the fact that markup-induced variation in the overall labor share is associated with an increase in the share of labor income going to these occupations. In contrast, the group represented by the blue triangles consists of the least N-intensive occupations and comprises approximately 24% of total labor income (machine operators, agriculture, transportation and construction). The relatively small N-component for these occupations is revealed by the fact that markup-induced variation in the overall labor share is associated with a decrease in the share of labor income going to these occupations. Figure 5 in Appendix E.2 shows very similar patterns for the reduced-form relationship between occupational income shares and the markup (adjusted for the negative relationship between the inverse labor share and the markup), and for the OLS relationship between occupational income shares and the de-trended inverse labor share (albeit with more noise since more variation in the labor share is being used).

4.3 Estimates of Occupational Factor Shares

Baseline Estimates We estimate $\{\eta_{jY}, \eta_{jN}\}_{j=1}^{J}$ using a GMM estimator, based on the moment conditions using each of the three types of variation described in Section 4.1. The results are displayed in Table 3. We set $(\gamma_Y, \gamma_N, \mu) = (0.620, 0.804, 1.2)$ based on the estimates in Table 1. Appendix E.2 contains estimates from alternative specifications for these first step parameters.

Our estimates using the de-trended markup as an instrument for the de-trended inverse labor share are shown in Panel A of Table 3. The occupations are ordered from the most N-intensive to the least N-intensive, as measured by the share of occupational labor income that compensates N-type activities, $\frac{S_{jN}}{S_j} = \frac{\eta_{jN} S_{LN}}{S_j}$. As anticipated by Figure 2b, there is heterogeneity across occupations, with N-content shares ranging from 27% for high-tech occupations to 11% for construction, extractive occupations and farming. It is striking that the ranking of occupations lines up with traditional notions of white-collar versus blue-collar occupations, yet these estimates were obtained entirely from the relative co-movement of occupational income shares with markups. No prior knowledge of the tasks that these occupations actually do was used in constructing this ranking.

The relatively small, but statistically significant, differences between η_{jN} and η_{jY}, manifest as large differences across occupations in their exposure to fluctuations in the overall labor share. This can be seen in the column labelled ε_{S_j,S_L}, which reports the elasticity of occupational income shares to the overall labor share, implied by the estimates of (η_{jY}, η_{jN}). The share-weighted average of these elasticities sums to one. High N-content occupations have an elasticity above one, whereas low N-content occupations have an elasticity below one (and even negative for some occupations).

The remaining panels of Table 3 report estimates without an instrument (Panel B) and using monetary shocks as an instrument (Panel C). The occupations are reported in the same order as in Panel A. In both cases, the results are very similar. The main difference is that when all of the cyclical variation in the labor share is used (Panel B), the additional variation leads to less precise estimates and less heterogeneity across occupations.

Industry Heterogeneity Our estimates of the N-intensity of different occupations uses business cycle variation in their relative labor income shares. Occupations whose labor

		η_Y	η_N	P-val $\eta_Y = \eta_N$	Elasticity ε_{S_j,S_L}	Share $\frac{S_{jN}}{S_j}$	P-val overid
Panel A: Instrument: De-trended Markup (IV)							
High Tech Occs		0.039	0.057	0.016	2.68	27%	
Service Occs		0.074	0.096	0.005	2.17	25%	
Managerial Occs		0.198	0.245	0.000	1.94	24%	
Admin Support, Clerical		0.111	0.128	0.108	1.63	23%	
Sales Occs		0.097	0.104	0.378	1.32	22%	
Professional Specialty		0.223	0.211	0.330	0.78	20%	
Production, Repair		0.071	0.053	0.002	-0.11	16%	
Machine Operators, Transportation		0.125	0.088	0.000	-0.28	16%	
Construction, Extractive, Farming		0.060	0.028	0.001	-1.49	11%	
First stage: R2	0.27						
First stage F	37.6						
Panel B: Instrument: De-trended Labor Share (OLS)							
High Tech Occs		0.042	0.045	0.459	1.22	21%	
Service Occs		0.078	0.081	0.343	1.19	21%	
Managerial Occs		0.204	0.219	0.015	1.29	22%	
Admin Support, Clerical		0.111	0.127	0.013	1.58	23%	
Sales Occs		0.097	0.104	0.055	1.33	22%	
Professional Specialty		0.223	0.213	0.142	0.81	20%	
Production, Repair		0.068	0.065	0.223	0.80	20%	
Machine Operators, Transportation		0.120	0.106	0.015	0.48	19%	
Construction, Extractive, Farming		0.054	0.048	0.198	0.53	19%	
Panel C: Instrument: Lagged Monetary Shocks (GMM)							
High Tech Occs		0.038	0.059	0.003	3.04	29%	0.436
Service Occs		0.075	0.088	0.020	1.67	23%	0.082
Managerial Occs		0.203	0.224	0.045	1.43	22%	0.151
Admin Support, Clerical		0.104	0.153	0.000	2.75	28%	0.134
Sales Occs		0.102	0.084	0.021	0.25	18%	0.780
Professional Specialty		0.219	0.226	0.445	1.13	21%	0.432
Production, Repair		0.072	0.052	0.000	-0.21	16%	0.522
Machine Operators, Transportation		0.126	0.083	0.000	-0.54	15%	0.673
Construction, Extractive, Farming		0.057	0.043	0.016	-0.05	16%	0.090
First stage: R2	0.36						
First stage F	8.69						

Table 3: Second step estimates of occupational factor share parameters.

This table shows the estimates for (η_{jN}, η_{jY}) in Equation (7).. Estimates in Panel A use de-trended markup as an instrument for de-trended inverse labor share. Estimates in Panel B use OLS. Estimates in Panel C use optimally-weighted GMM with contemporaneous and lagged monetary policy shocks at horizons of one to four quarters, from two series. See text for details of monetary policy shock series.

income share fluctuations are more strongly correlated with markup induced fluctuations in the overall labor share are interpreted through the lens of the model as having high N-intensity. One concern with this interpretation is that our simple model does not allow for heterogeneity across occupations in their usage in different industries. If the labor shares of some industries co-move more closely with markups at business cycle frequencies than other industries, then it may be that we identify some occupations as N-intensive simply because they are used more intensively in those industries. To investigate the role of industry composition, we estimate an extension of the model that allows for this form of industry heterogeneity. Full details of the model are in Appendix D. Here we describe the key features and the resulting estimating equations.

We assume that there are G different industries, indexed by g, and that final goods are a Cobb-Douglas aggregator of the value-added of each industry,

$$Y = \prod_{g=1}^{G} Y_g^{\varsigma_g}$$

where $\varsigma_g \in [0,1]$ are the value added-shares of each industry and satisfy $\sum_{g=1}^{G} \varsigma_g = 1$. Each industry is comprised of its own upstream and downstream sectors with industry-specific production and expansion labor elasticities, γ_{Yg} and γ_{Ng}. Heterogeneity in $(\gamma_{Yg}, \gamma_{Ng})$ across industries gives rise to heterogeneity in the cyclical co-movement between industry-level labor shares and markups. Production and expansion in each industry use labor from each occupation, but with different occupation shares in each industry

$$M_g = Z_{Ug} \left(\prod_{j=1}^{J} L_{jg,Y}^{\eta_{jgY}} \right)^{\gamma_{Yg}} \qquad N_g = Z_{Dg} \left(\prod_{j=1}^{J} L_{jg,N}^{\eta_{jgN}} \right)^{\gamma_{Ng}}$$

where the industry-specific occupation weights satisfy $\sum_{j=1}^{J} \eta_{jgY} = \sum_{j=1}^{J} \eta_{jgN} = 1 \; \forall g$. Thus, in this extended model, the occupational labor income shares of two different occupations j and j' may covary differently with markup-induced fluctuations in the overall labor share for two reasons. First, they may have different N-intensity within industries, i.e. $\eta_{jgY} - \eta_{jgN} \neq \eta_{j'gY} - \eta_{j'gN}$ for some industries g. Second, even if they have the same N-intensity within industries, i.e. $\eta_{jgY} = \eta_{jgN}$ and $\eta_{j'gY} = \eta_{j'gN}$ for all g, they may have different overall usage in industries that have different exposure to markups, i.e. $\eta_{jg} \neq \eta_{j'g}$ for some industries g,g' with $\gamma_{Yg} - \gamma_{Ng} \neq \gamma_{Yg'} - \gamma_{Ng'}$. By estimating the industry-level production and expansion elasticities, and the industry-specific occupation weights, we can separate these two forces.

The empirical specifications for the first step estimation are the industry-level analogues of equation (2):

$$S_{Lg,t} = \gamma_{Ng} + (\gamma_{Yg} - \gamma_{Ng}) \frac{1}{\mu_{gt}} + \epsilon_{Lg,t} \tag{12}$$
$$\frac{1}{\mu_{gt}} = \frac{1}{\mu_t} + \epsilon_{g,t}$$
$$\frac{1}{\mu_t} = \frac{1}{\mu} + \epsilon_{\mu,t}$$

We assume that industry-level markups μ_{gt} are comprised of a common average markup plus a common time-varying component and an industry-specific time-varying component. The assumption required for identification of $(\gamma_{Yg}, \gamma_{Ng})$ is that at business cycle frequencies, variation in the common component of industry-level inverse markups $\epsilon_{\mu,t}$ is independent of both industry-specific inverse markup variation $\epsilon_{g,t}$ and other sources of cyclical variation in industry labor shares that are outside the model, as well as measurement error in industry labor shares (captured by $\epsilon_{Lg,t}$). The required moment conditions are

$$E\left[\epsilon_{Lg,t}\right] = 0 \ \forall (t, g)$$
$$E\left[\epsilon_{Lg,\tau} \middle| \epsilon_{\mu,t}, \epsilon_{g,t}\right] = 0 \ \forall (\tau, t, g).$$

We use annual data from the Bureau of Economic Analysis' Integrated Industry-Level Production Accounts (KLEMS) on detrended industry-level labor shares for ten super-sectors (excluding government) for 1987-2019 . We estimate equation (2) by GMM, subject to the constraints that $\gamma_Y, \gamma_N \leq 1$.[13] As in our baseline specification, we use the de-trended price index ratio ϱ_t as a proxy for the markup and set the average markup over the period to 1.2. Our estimates are contained in Table 4. There are substantial differences across industries in both the overall labor share and the share of labor payments that compensate N-type activities. Finance, which is the largest industry by value added, has both the lowest overall labor share (26%) and the lowest share of N-type labor (16%). The industries with the largest share of N-type labor are Resources and Mining (35%), Manufacturing (24%), and Information (24%), which are all industries with relatively low overall labor shares.

The empirical specification for the second step is the industry-level analogue of equation (7),

$$s_{jg,t} = \eta_{jYg} + (\eta_{jgN} - \eta_{jgY}) \left(1 - \frac{\gamma_{Yg}}{\gamma_{Ng}}\right)^{-1} \left(1 - \gamma_{Yg} \frac{1}{S_{Lg,t}}\right) + \epsilon_{s_{jg},t} \ \forall j, g \tag{13}$$

[13] In Appendix (E.2), we report results using data from from the World KLEMS Database for 1950-2014, and estimates without the constraint that $\gamma_Y, \gamma_N \leq 1$.

	γ_{Yg}	γ_{Ng}	P-val test for $\gamma_{Yg} = \gamma_{Ng}$	Implied $\frac{S_{LNg}}{S_{Lg}}$	Mean S_{Lg}	Mean share of value added
Resources and Mining	0.365	1.000	0.000	35.4%	47.7%	3.1%
Construction	0.814	1.000	0.000	19.7%	84.5%	4.8%
Manufacturing	0.512	0.797	0.000	23.7%	55.9%	16.3%
Trade, Transp., Util.	0.534	0.729	0.000	21.4%	56.7%	19.9%
Information	0.373	0.588	0.137	24.0%	40.8%	5.6%
Finance	0.259	0.239	0.860	15.6%	25.5%	22.3%
Prof. and Business Serv.	0.795	0.960	0.080	19.4%	82.2%	12.3%
Educ. and Health Serv.	0.876	0.948	0.315	17.8%	88.8%	8.7%
Leisure and Hospitality	0.678	0.747	0.359	18.1%	69.0%	4.3%
Other Services	0.785	0.847	0.699	17.8%	79.5%	2.8%

Table 4: First step estimation results, industries. BEA KLEMS annual data 1987-2019. Assumed mean markup, $\mu = 1.2$.

where $s_{jg,t}$ are occupational labor income shares within industry g. We estimate this system of equation with GMM, using the de-trended price ratio as an instrument. We use annual data from KLEMS for 1987 to 2019. Our findings are reported in Table 5. For comparability with our baseline estimates that use quarterly data, Column (1) (labelled "No industries") contains estimates from the model with a single industry, but using annual data. Column (2) (labelled "Industries") contains estimates of equation (13). Column (3) (labelled "Industries restricted") is our preferred industry specification. In this specification we re-parameterize (η_{jgY}, η_{jgN}) as

$$\chi_{jg} := \eta_{jgN} + \eta_{jgY}, \quad \alpha_{jg} := \frac{\eta_{jgY}}{\eta_{jgN} + \eta_{jgY}},$$

and assume that $\alpha_{jg} = \alpha_j$ for g. This has the effect of allowing different occupations to have different relative importance in each industry (χ_{jg}) while imposing that the relative N-intensity of an occupation does not depend on the industry in which it is being used. To calculate shares of the overall labor income in each occupation that compensates N-type activities, we use the average value- added shares labor shares across industries industries.

Table 5 indicates that part of the heterogeneity in the co-movement of occupational labor shares with markup-induced moments in the overall labor share is indeed due to industry heterogeneity. However, taking industries into account, substantial heterogeneity remains. High-tech occupations continue to be the most N-intensive (27%), while Service occupations are now the least N-intensive (15%). Among the remaining occupations, the differences in N-intensity are smaller than in the version without industry heterogeneity.

	(1) No industries	(2) Industries	(3) Industries restricted
	Share $\frac{S_{jN}}{S_j}$	Share $\frac{S_{jN}}{S_j}$	Share $\frac{S_{jN}}{S_j}$
High Tech Occs	26%	24%	27%
Service Occs	20%	18%	15%
Managerial Occs	23%	19%	19%
Admin Support, Clerical	24%	20%	27%
Sales Occs	22%	30%	19%
Professional Specialty	20%	19%	21%
Production, Repair	15%	18%	19%
Machine Operators, Transportation	14%	18%	19%
Construction, Extractive, Farming	14%	21%	21%

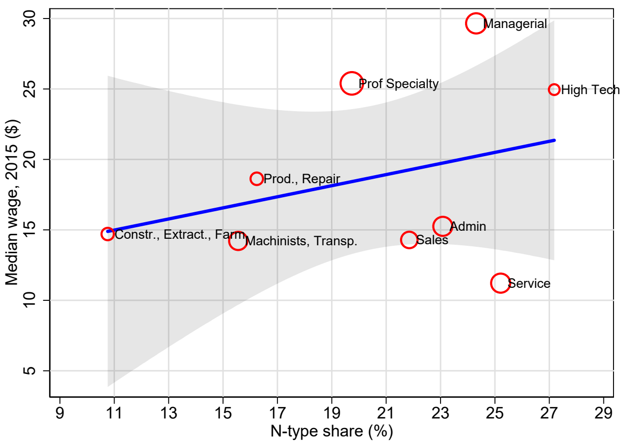
Table 5: Second step estimates of occupational factor share parameters with industry heterogeneity.
This table shows the implied S_{jN}/S_j ratios. In column (1) we use estimates these from the model with a single industry. In column (2) we show results obtained by using estimates for (η_{jgN}, η_{jgY}) in Equation (13), estimated for each (j, g) separately.. Results in column (3) use estimates for (η_{jgN}, η_{jgY}) obtained by imposing that the relative N-intensity of an occupation does not depend on the industry in which it is being used. See text for details.

4.4 Characteristics of N-intensive Occupation

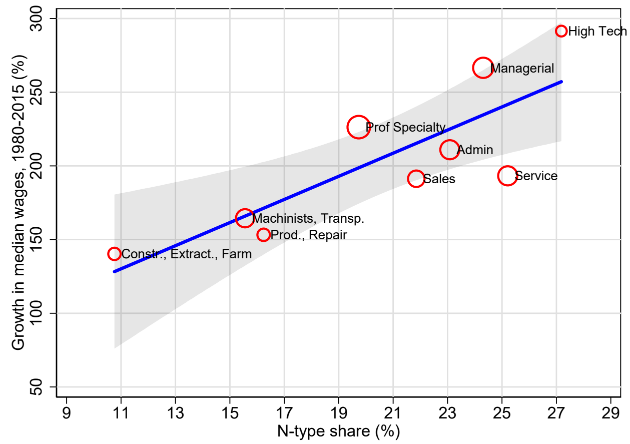
Having estimated the extent to which workers in different occupations are engaged in production activities versus expansionary activities, in this section we explore the characteristics of these different occupations.

We start by using the 1980 Census and 2015 American Community Survey (ACS) to measure total hours and median hourly wages for each of the nine occupation groups. Figure 3a shows a scatter plot of median hourly wages in 2015 against the N-content share of each occupation group. There is only a weak relationship between the level of wages and N-content. Although the high-wage occupation groups are mostly high N-content occupations (managerial, high-tech), there are also low-wage occupations with high N-content (service, admin), and the low N-content occupations are in the middle of the distribution. (Viewed on its side, Figure 3a suggests a U-shaped relationship between N-content and wages).

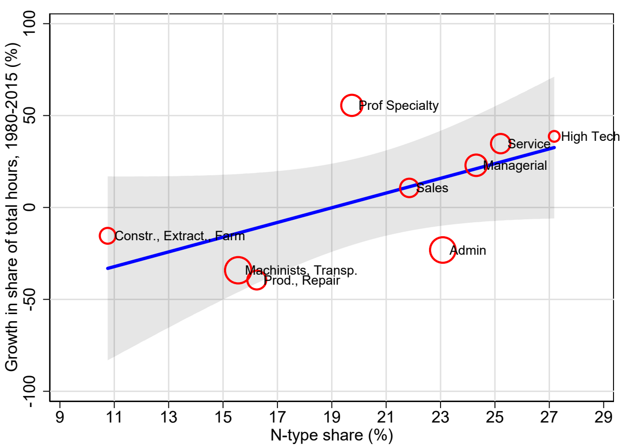
Wage growth, on the other hand, is strongly correlated with N-content. This can be seen in Figure 3b, which plots the cumulative nominal growth in median wages from 1980 to 2015 in each occupation against the N-share. Figure 3c also shows a positive correlation between N-content and the growth in the share of total hours from 1980 to 2015. The fact that growth has been strongest in both the quantity and price of occupations with high



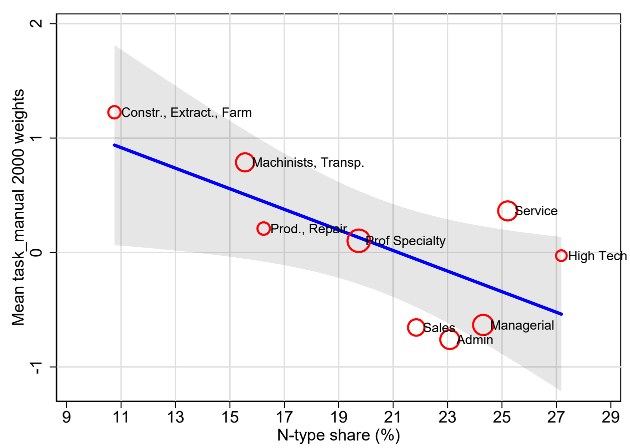
(a) Median wages 2015



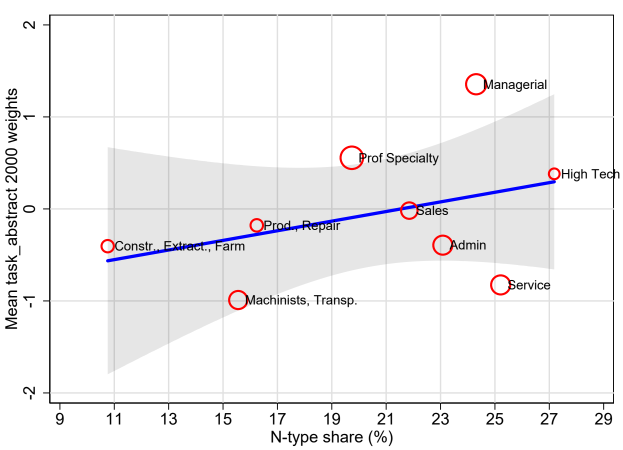
(b) Median wage growth, 1980-2015



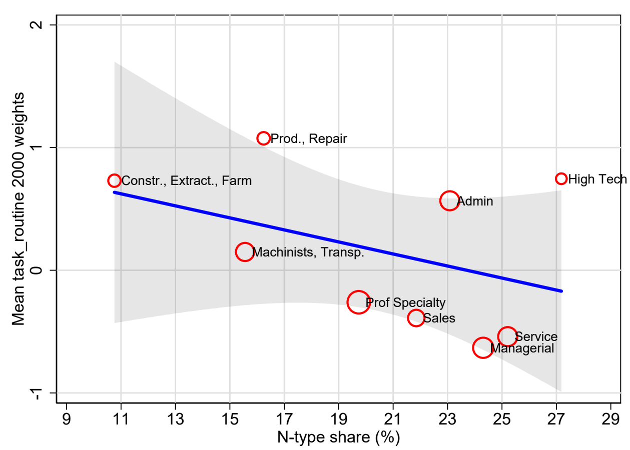
(c) Growth in share of total hours, 1980-2015



(d) Manual content, 2000 weights



(e) Abstract content, 2000 weights



(f) Routine content, 2000 weights

Figure 3: Correlation of N-content of occupations with other occupation characteristics. Wage and hours data from 1980 Census and 2015 American Community Survey. Routine corresponds to average of DOT measures: "set limits, tolerances and standards," and "finger dexterity." Manual corresponds to DOT measure "eye-hand-foot coordination". Abstract is average of DOT measures: "direction, control and planning" and "GED math." See Autor et al. (2006) for details.

N-content, suggest that labor demand for expansionary activities has increased faster than labor demand for traditional production activities. .

The remaining three panels of Figure 3 show how the estimated N-content of each occupation correlates with the three broad task measures constructed by Autor et al. (2006) from the US Labor Department's Dictionary of Occupational Titles (DOT).[14] These figures suggest that N-content is negative correlated with the manual content of occupations (as reflected in the DOT measure "eye-hand-foot coordination"), weakly positively correlated with the abstract content of occupations (as reflected in the DOT measures "direction, control and planning" and "GED math"), and weakly negatively correlated with the routine content of occupations (as reflected in the DOT measures "set limits, tolerances and standards," and "finger dexterity").

5 Conclusion

We have demonstrated that the distinction between two uses of labor in modern economies – expansionary, or N-type, activities, versus production, or Y-type, activities – is critical for understanding the relationship between markups and the labor income distribution. We developed a framework that operationalizes this distinction at the occupational level. Estimation using post-war US data at business cycle frequencies suggest that around one-fifth of total US labor income compensates N-type activities, and reveals substantial heterogeneity across occupations in their degree of N-intensity. Those occupations with the largest expansionary content are those that are typically labelled as white-collar occupations, while those with the least expansionary content are those that are typically labelled as blue-collar occupations. When markups rise, labor income shifts away from the Y-intensive occupations and toward N-intensive occupations.

Future research can extend our work in several fruitful directions. These include allowing for firm-level heterogeneity in markups and production and expansion elasticities; investigating changes over time in the level of markups, production and expansion elasticities and the relative productivity of expansion versus production; incorporating the distinction between Y-type and N-type labor into settings with heterogeneous households to study the distributional effects of changes in markups at the household level; and incorporating our structure into representative agent and heterogeneous agent New Keynesian models, as a mechanism or generating pro-cyclical profits and a counter-cyclical labor share.

[14]These measures are standardized to have a mean of 0 and standard deviation of 1. They are aggregated from detailed occupation groups using 2000 Census weights. In Figure 6 in Appendix E.2 we present analogous figures using the six Work Context and Work Activity measures from O*NET as defined in Acemoglu and Autor (2011).

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A Proofs and Derivations

A.1 Proof of Lemma 1

Proof. The upstream firm solves

$$\Pi_U := \max_{M,\{L_{jY}\}_{j=1}^{J}} P_U M - \sum_{j=1}^{J} W_j L_{jY}$$
$$\text{subject to}$$
$$M = Z_U \left(\prod_{j=1}^{J} L_{jY}^{\eta_{jY}} \right)^{\gamma_Y}$$

and the first order condition with respect to L_{kY} is

$$W_k = P_U \gamma_Y \eta_{kY} Z_U \left(\prod_{j=1}^{J} L_{jY}^{\eta_{jY}} \right)^{\gamma_Y - 1} \frac{\prod_{j=1}^{J} L_{jY}^{\eta_{jY}}}{L_{kY}}.$$

Use $M = Z_U \left(\prod_{j=1}^{J} L_{jY}^{\eta_{jY}} \right)^{\gamma_Y}$ to write it as

$$W_k = P_U \gamma_Y \eta_{kY} \frac{M}{L_{kY}}$$

and use the fact that in symmetric equilibrium $p = \mu P_U$ to rewrite it as

$$W_k = \frac{p}{\mu} \gamma_Y \eta_{kY} \frac{M}{L_{kY}} \tag{14}$$

which can be rearranged as (using market clearing $M = Nm = Ny = Y$)

$$\frac{W_k L_{kY}}{pY} = \gamma_Y \eta_{kY} \frac{1}{\mu}.$$

This shows that

$$\begin{aligned} S_{LY} &= \frac{\sum_{j=1}^{J} W_j L_{jY}}{pY} \\ &= \sum_{j=1}^{J} \gamma_Y \eta_{kY} \frac{1}{\mu} \\ &= \gamma_Y \frac{1}{\mu}. \end{aligned}$$

The downstream firm solves

$$\Pi_D := \max_{N, \{L_{jN}\}_{j=1}^J} \int_0^N \Pi_i di - \sum_{j=1}^J W_j L_{jN}$$

subject to

$$N = Z_D \left(\prod_{j=1}^J L_{jN}^{\eta_{jN}} \right)^{\gamma_N}$$

which, in a symmetric equilibrium with $\Pi_i = \Pi$ for all i, can be written as

$$\Pi_D := \max_{N, \{L_{jN}\}_{j=1}^J} N\Pi - \sum_{j=1}^J W_j L_{jN}$$

subject to

$$N = Z_D \left(\prod_{j=1}^J L_{jN}^{\eta_{jN}} \right)^{\gamma_N}$$

and the first order condition with respect to L_{kN} is

$$W_j = \gamma_N \eta_{kN} \frac{N}{L_{kN}} \Pi.$$

In a symmetric equilibrium

$$\Pi = py \left(1 - \frac{1}{\mu}\right).$$

Use this fact in the first order condition to get

$$W_k = \gamma_N \eta_{kN} \frac{N}{L_{kN}} py \left(1 - \frac{1}{\mu}\right)$$

and recall market clearing $M = Nm = Ny = Y$ to write

$$W_k = \gamma_N \eta_{kN} \frac{pY}{L_{kN}} \left(1 - \frac{1}{\mu}\right) \tag{15}$$

which can be rearranged as

$$\frac{W_k L_{kN}}{pY} = \gamma_N \eta_{kN} \left(1 - \frac{1}{\mu}\right).$$

This shows that

$$\begin{aligned} S_{LN} &= \frac{\sum_{j=1}^J W_j L_{jN}}{pY} \\ &= \gamma_N \left(1 - \frac{1}{\mu}\right) \sum_{j=1}^J \eta_{jN} \\ &= \gamma_N \left(1 - \frac{1}{\mu}\right). \end{aligned}$$

Finally,

$$
\begin{aligned}
S_L &= S_{LN} + S_{LY} \\
&= \gamma_Y \frac{1}{\mu} + \gamma_N \left(1 - \frac{1}{\mu}\right).
\end{aligned}
$$

To get the share of total income accruing to occupation j notice that

$$
W_j L_j = \left(1 - \frac{1}{\mu}\right) \gamma_N \eta_{jN} pY + \frac{1}{\mu} \gamma_Y \eta_{jY} pY
$$

so

$$
\begin{aligned}
S_j &:= \frac{W_j L_j}{pY} \\
&= \frac{\left(1 - \frac{1}{\mu}\right) \gamma_N \eta_{jN} pY + \frac{1}{\mu} \gamma_Y \eta_{jY} pY}{pY} \\
&= \left(1 - \frac{1}{\mu}\right) \gamma_N \eta_{jN} + \frac{1}{\mu} \gamma_Y \eta_{jY}.
\end{aligned}
$$

It can also be written as

$$
S_j = \eta_{jY} S_{LY} + \eta_{jN} S_{LN}.
$$

To obtain profit shares note that

$$
\begin{aligned}
S_D &:= \frac{\Pi_D}{pY} \\
&= \frac{pNy\left(1 - \frac{1}{\mu}\right) - \sum_{j=1}^{J} W_j L_{jN}}{pY} \\
&= \left(1 - \frac{1}{\mu}\right) - \gamma_N \left(1 - \frac{1}{\mu}\right) \\
&= (1 - \gamma_N) \left(1 - \frac{1}{\mu}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
S_U &:= \frac{\Pi_U}{pY} \\
&= \frac{P_U M - \sum_{j=1}^{J} W_j L_{jY}}{pY} \\
&= \frac{1}{\mu} - S_{LY} \\
&= (1 - \gamma_Y) \frac{1}{\mu}
\end{aligned}
$$

so

$$
\begin{aligned}
S_\Pi &= S_U + S_D \\
&= (1 - \gamma_Y) \frac{1}{\mu} + (1 - \gamma_N) \left(1 - \frac{1}{\mu}\right)
\end{aligned}
$$

□

A.2 Proof of Theorem 1

Proof. As shown in Lemma 1

$$S_{LY} = \gamma_Y \frac{1}{\mu}$$
$$S_{LN} = \gamma_N \left(1 - \frac{1}{\mu}\right)$$

so

$$\frac{\partial S_{LY}}{\partial \mu} = -\gamma_Y \frac{1}{\mu^2} < 0$$
$$\frac{\partial S_{LN}}{\partial \mu} = \gamma_N \frac{1}{\mu^2} > 0.$$

□

A.3 Proof of Theorem 2

Proof. As shown in Lemma 1

$$S_j = \eta_{jY} S_{LY} + \eta_{jN} S_{LN}$$

so

$$\begin{aligned} s_j &= \eta_{jY} \frac{\gamma_Y \frac{1}{\mu}}{\gamma_Y \frac{1}{\mu} + \gamma_N \left(1 - \frac{1}{\mu}\right)} + \eta_{jN} \frac{\gamma_N \left(1 - \frac{1}{\mu}\right)}{\gamma_Y \frac{1}{\mu} + \gamma_N \left(1 - \frac{1}{\mu}\right)} \\ &= \eta_{jY} + (\eta_{jN} - \eta_{jY}) \left[\frac{(\mu - 1)\,\gamma_N}{\gamma_Y + (\mu - 1)\,\gamma_N}\right] \end{aligned}$$

with

$$\frac{\partial s_j}{\partial \mu} = (\eta_{jN} - \eta_{jY}) \frac{\partial}{\partial \mu} \left[\frac{(\mu - 1)\,\gamma_N}{\gamma_Y + (\mu - 1)\,\gamma_N}\right].$$

Since $\frac{\partial}{\partial \mu}\left[\frac{(\mu-1)\gamma_N}{\gamma_Y+(\mu-1)\gamma_N}\right] \geq 0$ (> 0 if $\gamma_N > 0$) we have

$$\frac{\partial s_j}{\partial \mu} \geq 0$$

if and only if

$$\gamma_N \geq \gamma_Y.$$

□

A.4 Proof of Theorem 3

As shown in Lemma 1

$$S_L = \gamma_Y \frac{1}{\mu} + \gamma_N \left(1 - \frac{1}{\mu}\right)$$

with

$$\frac{\partial S_L}{\partial \mu} = \frac{1}{\mu^2}\left(\gamma_N - \gamma_Y\right)$$

which is positive if and only if

$$\gamma_N > \gamma_Y.$$

Similarly, since

$$S_\Pi = 1 - S_L$$

we have

$$\frac{\partial S_\Pi}{\partial \mu} = -\frac{1}{\mu^2}\left(\gamma_N - \gamma_Y\right)$$

which is positive if and only if

$$\gamma_N < \gamma_Y.$$

B Details of Alternative Preference and Market Structures

This Appendix provides further details of the preference and market structures referred to in Section 2.4.

B.1 Demand System Preliminaries

Definitions We define the following objects:

- Ω is the measure of unique varieties being produced in the economies. $\omega \in [0, \Omega]$ are individual varieties. p_ω is the price faced by consumers for variety ω.
- N the measure of establishments or retail sales units. Some varieties might be produced by more than one retail sales unit but each sales units produces only one variety. $i \in [0, N]$ are individual retail sales units. p_i is the price charged by sales unit i. y_i is the quantity sold by sales unit i. μ_i is the markup over marginal charged by sales unit i.
- $\mathcal{M} := \frac{N}{\Omega}$ is the measures of sales units producing each variety. We assume that when a sales unit produces a new variety it is chosen randomly, so that the same measure of firms operate in each variety.

Households Households choose c_ω given prices p_ω. Households have utility defined over an aggregator of varieties $C\left(\{c_\omega\}_{\omega\in[0,\Omega]},\Omega\right)$. The household solves the following problem:

$$V\left(\{p_\omega\}_{\omega\in[0,\Omega]},I,\Omega\right)=\max_{c_\omega}U\left(C\left(\{c_\omega\}_{\omega\in\Omega},\Omega\right)\right)$$
$$\text{subject to}$$
$$I\geq\int_0^\Omega p_\omega c_\omega d\omega$$

where $V(\bullet)$ is the indirect utility function and $U(\bullet)$ is the direct utility function which is assumed to be strictly increasing. The household as income I.

Definition of love-of-variety Consider a household with income I. Assume that $p_\omega = p\forall\omega$ and that a household purchases the same quantity $c_\omega = c$ of each good, meaning they allocate expenditure equally across the goods. Define the indirect utility associated with this pattern of expenditure as $V(p,I,\Omega)$. We say that the demand system features love-of-variety if $\frac{\partial V(p,I,\Omega)}{\partial\Omega}>0$ and no love-of-variety if $\frac{\partial V(p,I,\Omega)}{\partial\Omega}=0$. Note that this in this symmetric case

$$I=\Omega pc$$

and the indirect utility function is a monotonic function of $C(c,\Omega)=C\left(\frac{I}{\Omega p},\Omega\right)$. So the condition for no-love-of-variety is equivalent to

$$-\frac{c}{\Omega}\frac{\partial C}{\partial c}+\frac{\partial C}{\partial\Omega}=0$$
$$\varepsilon_{c,\Omega}=1$$

i.e. the elasticity of substitution between c and Ω is equal to 1. To a first-order this implies that

$$C=c\Omega$$

Definition of price index Recall that definition of an expenditure function

$$E\left(\{p_\omega\}_{\omega\in[0,\Omega]},C,\Omega\right)=\min_{c_\omega}\int_0^\Omega p_\omega c_\omega d\omega$$
$$\text{subject to}$$
$$C\left(\{c_\omega\}_{\omega\in\Omega},\Omega\right)\geq C$$

For homothetic preferences, meaning that the aggregator is homogenous of degree 1, the price index P is defined as the minimum cost of obtaining one unit of the bundle C:

$$P\left(\{p_\omega\}_{\omega\in[0,\Omega]},\Omega\right)=E\left(\{p_\omega\}_{\omega\in[0,\Omega]},1,\Omega\right)$$

and the expenditure function takes the form

$$E\left(\{p_\omega\}_{\omega\in[0,\Omega]},C,\Omega\right)=P\left(\{p_\omega\}_{\omega\in[0,\Omega]},\Omega\right)C$$

If presences are not homothetic, then we can define a price index that depends on the level of the consumption bundle as

$$P\left(\{p_\omega\}_{\omega\in[0,\Omega]},C,\Omega\right)=\frac{E\left(\{p_\omega\}_{\omega\in[0,\Omega]},C,\Omega\right)}{C}$$

Market clearing The total measure of variety ω sold must equal the measure of variety ω consumed

$$
\begin{aligned}
c_\omega &= \frac{N}{\Omega} y_i = \mathcal{M} y_i \\
c\Omega &= Ny \\
C &= Ny
\end{aligned}
$$

where the last line follows from no love- of variety. In the more general case, we would have

$$C^{-1}(C, \Omega)\,\Omega = Ny$$

Typically this takes the form

$$Cf(\Omega) = c\Omega$$

So the market clearing condition gives

$$
\begin{aligned}
c_\omega &= \frac{N}{\Omega} y_i = \mathcal{M} y_i \\
c\Omega &= Ny \\
Cf(\Omega) &= Ny
\end{aligned}
$$

Symmetric equilibria We will focus on symmetric demand systems and symmetric equilibria. This means that the price index takes the form $P(p, C, \Omega)$ or $P(p, \Omega)$ in the case of homothetic preferences. In the case of homothetic preferences, we can express the love of variety condition in terms of the price index. With $p_\omega = p \forall \omega$ and $c_\omega = c$, the expenditure function and definition of the price index imply

$$\Omega pc = P(p, \Omega)\, C$$

the condition for no-love-of-variety is then

$$
\begin{aligned}
\Omega pc &= P(p, \Omega)\, c\Omega \\
p &= P(p, \Omega)
\end{aligned}
$$

which implies that the price index satisfies $P = p$ and does not depend on Ω.

Demand functions The demand functions solve

$$c_\omega\left(\{p_\omega\}_{\omega\in[0,\Omega]}, I, \Omega\right) = \max_{c_\omega} U\left(C\left(\{c_\omega\}_{\omega\in\Omega}, \Omega\right)\right)$$

$$\text{subject to}$$

$$I \geq \int_0^\Omega p_\omega c_\omega d\omega$$

using the definition of the expenditure function, i.e that $I = P\left(\{p_\omega\}_{\omega\in[0,\Omega]}, C, \Omega\right) C$ we can write these as $c_\omega\left(\{p_\omega\}_{\omega\in[0,\Omega]}, C, \Omega\right)$, With homothetic preferences, these take the form $c_\omega = f\left(\{p_\omega\}_{\omega\in[0,\Omega]}, P, \Omega\right) C$. The elasticity of demand is denoted by

$$\varepsilon = -\frac{p_\omega}{c_\omega}\frac{\partial c_\omega}{\partial p_\omega}$$

B.2 Demand Systems

B.2.1 CES

The aggregator function is

$$C = \left[\Omega^{-\frac{\rho}{\sigma}} \int_0^\Omega c_\omega^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

In a symmetric equilibrium this gives

$$C = \Omega^{\frac{\sigma-\rho}{\sigma-1}} c$$

so the preferences feature no love of variety if $\rho = 1$. The preferences are homothetic because the aggregator is homogenous of degree 1 in c.

The price index is

$$P = \left[\Omega^{-\rho} \int_0^\Omega p_\omega^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

which gives in a symmetric equilibrium

$$P = \Omega^{\frac{1-\rho}{1-\sigma}} p$$

The demand functions are

$$c_\omega = \left(\frac{p_\omega}{P} \right)^{-\sigma} C \Omega^{-\rho}$$

so the elasticity of demand is

$$\varepsilon = \sigma$$

B.2.2 Translog

There is no closed form expression for the aggregator but the preferences are homothetic.

The price index is given by

$$\log P = \frac{1}{2\sigma\Omega} + \frac{1}{\Omega} \int_0^\Omega \log p_\omega d\omega + \frac{1}{2} \int_0^\Omega \int_0^\Omega \frac{\sigma}{\Omega} \log p_\omega \left(\log p_{\omega'} - \log p_\omega \right) d\omega d\omega'$$

In a symmetric equilibrium this gives

$$\begin{aligned} \log P &= \frac{1}{2\sigma\Omega} + \log p \\ \log\left(\frac{P}{p}\right) &= \frac{1}{2\sigma\Omega} \\ P &= p e^{\frac{1}{2\sigma\Omega}} \end{aligned}$$

so this features love of variety.

The demand function is

$$\begin{aligned}
c_\omega &= \left[\frac{1}{\Omega} - \sigma\left(\log p_\omega - \frac{1}{\Omega}\int_0^\Omega \log p_{\omega'} d\omega'\right)\right]\frac{I}{p_\omega} \\
&= \left[\frac{1}{\Omega} - \sigma\left(\log p_\omega - \log p\right)\right]\frac{I}{p_\omega} \\
&= \left[\frac{1}{\Omega} - \sigma\left(\log p_\omega - \log P + \frac{1}{2\sigma\Omega}\right)\right]\frac{PC}{p_\omega} \\
\log c_\omega &= \log\left[\frac{1}{\Omega} - \sigma\left(\log p_\omega - \log P + \frac{1}{2\sigma\Omega}\right)\right] + \log P + \log C - \log p_\omega
\end{aligned}$$

where the second line follows from symmetry and the third line follow form the fact that preferences are homothetic. In a symmetric equilibrium this implies

$$\begin{aligned}
\log c &= \log\frac{1}{\Omega} + \frac{1}{2\sigma\Omega} + \log C \\
C &= c\Omega e^{-\frac{1}{2\sigma\Omega}}
\end{aligned}$$

which does not give $C = c\Omega$ because of the love of variety.

The elasticity of demand is

$$\begin{aligned}
\varepsilon &= \frac{\sigma}{\frac{1}{\Omega} - \sigma\left(\log p_\omega - \log P + \frac{1}{2\sigma\Omega}\right)} + 1 \\
&= \frac{\sigma}{\frac{1}{\Omega} - \sigma\left(\log p - \log P + \frac{1}{2\sigma\Omega}\right)} + 1 \\
&= \frac{\sigma}{\frac{1}{\Omega}} + 1 \\
&= \sigma\Omega + 1
\end{aligned}$$

B.2.3 Linear Demand

The aggregator is

$$C = \int_0^\Omega c_\omega d\omega - \frac{1}{2\sigma}\int_0^\Omega c_\omega^2 d\omega + \frac{1}{\Omega 2\sigma}\left[\int_0^\Omega c_\omega d\omega\right]^2$$

which gives in a symmetric equilibrium

$$C = \Omega c$$

So in a symmetric equilibrium they do not feature love of variety.

Are preferences homothetic?

$$\int_0^\Omega t c_\omega d\omega - \frac{1}{2\sigma}\int_0^\Omega (tc_\omega)^2 d\omega + \frac{1}{\Omega^{1\rho}2\sigma}\left[\int_0^\Omega (tc_\omega)\, d\omega\right]^2 = t\int_0^\Omega c_\omega d\omega - \frac{t^2}{2\sigma}\int_0^\Omega c_\omega^2 d\omega + \frac{t^2}{\Omega 2\sigma}\left[t\int c_\omega d\omega\right]^2$$

which equals tC only in the symmetric case, so no, because homotheticity requires this to equal tC even in the non-symmetric case.

We can derive the price index as

$$P = \min_{c_\omega} \int_0^\Omega c_\omega p_\omega d\omega$$
$$\text{subject to}$$
$$1 \leq \int_0^\Omega c_\omega d\omega - \frac{1}{2\sigma} \int_0^\Omega c_\omega^2 d\omega + \frac{1}{2\sigma} \left[\int_0^\Omega c_\omega d\omega \right]^2$$

which gives

$$p_\omega = \lambda \left[-1 + \frac{1}{\sigma} c_\omega - \frac{1}{\Omega\sigma} \int_0^\Omega c_{\omega'} d\omega' \right]$$
$$c_\omega = \sigma \left(\frac{p_\omega}{\lambda} + 1 \right) + \frac{1}{\Omega} \int_0^\Omega c_{\omega'} d\omega'$$

In a symmetric equilibrium

$$c_\omega = \sigma \left(\frac{p}{\lambda} + 1 \right) + c$$

and substituting into the constraint at equality

$$1 = \int_0^\Omega \left[\sigma \left(\frac{p}{\lambda} + 1 \right) + c \right] d\omega$$
$$\frac{p}{\lambda} + 1 = \frac{1 - \Omega c}{\Omega \sigma}$$

so

$$c_\omega = \frac{1}{\Omega}$$
$$P = \frac{1}{\Omega} \int_0^\Omega p_\omega d\omega$$

which gives

$$P = p$$

in a symmetric equilibrium

The demand function is given by

$$\max_{c_\omega} \int_0^\Omega c_\omega d\omega - \frac{1}{2\sigma} \int_0^\Omega c_\omega^2 d\omega + \frac{1}{\Omega 2\sigma} \left[\int_0^\Omega c_\omega d\omega \right]^2$$
$$\text{subject to}$$
$$\int_0^\Omega p_\omega c_\omega d\omega = I$$

which gives

$$\lambda p_\omega = 1 - \frac{1}{\sigma}c_\omega + \frac{1}{\Omega\sigma}\int_0^\Omega c_\omega d\omega$$
$$\lambda\int_0^\Omega p_\omega d\omega = \int_0^\Omega \left[1 - \frac{1}{\sigma}c_\omega + \frac{1}{\Omega\sigma}\int_0^\Omega c_\omega d\omega\right] d\omega$$
$$\lambda\Omega p = \Omega - \frac{1}{\sigma}\Omega c + \frac{1}{\sigma}\int_0^\Omega c_\omega d\omega$$

Dividing

$$\frac{p_\omega}{\Omega p} = \frac{1 - \frac{1}{\sigma}c_\omega + \frac{1}{\Omega\sigma}\int_0^\Omega c_\omega d\omega}{\Omega - \frac{1}{\sigma}\Omega c + \frac{1}{\sigma}\int_0^\Omega c_\omega d\omega}$$
$$= \frac{1 - \frac{1}{\sigma}c_\omega + \frac{1}{\sigma}c}{\Omega}$$
$$c_\omega = \frac{C}{\Omega} + \sigma\left(1 - \frac{p_\omega}{P}\right)$$

The elasticity of demand is

$$\varepsilon = \frac{\sigma}{P}\frac{p_\omega}{c_\omega}$$
$$= \sigma\frac{\Omega}{C}$$

B.2.4 Kimball

The aggregator C is defined implicitly by

$$\frac{1}{\Omega}\int_0^\Omega \Upsilon\left(\frac{\Omega c_\omega}{C}\right) d\omega = 1$$

where Υ satisfies $\Upsilon(1) = 1$. In a symmetric equilibrium this implies $C = c\Omega$

The demand function is given by

$$c_\omega = \frac{C}{\Omega}\Upsilon'^{-1}\left(\frac{p_\omega}{P}D\right)$$

where P is a price index defined by

$$PC = \int_0^\Omega p_\omega c_\omega d\omega$$

and D is a demand index defined by

$$D = \int_0^\Omega \frac{c_\omega}{C}\Upsilon'\left(\frac{\Omega c_\omega}{C}\right) d\omega$$

In a symmetric equilibrium, the demand index is just

$$D = \frac{\Omega c}{C}\Upsilon'\left(\frac{\Omega c}{C}\right)$$

They propose the following functional forms

$$\Upsilon'(x) = \frac{\sigma - 1}{\sigma} \exp\left\{\frac{1 - x^{\frac{\eta}{\sigma}}}{\eta}\right\}$$

which elasticity of demand

$$\varepsilon = \sigma \left(\frac{\Omega c_\omega}{C}\right)^{-\frac{\eta}{\sigma}}$$

which equals σ in a symmetric equilibrium.

B.3 Market Structures

Under each of the following market structures, the factor shares take the same form as in Lemma 1, where the (possibly endogenous) markup μ is given as follows.

B.3.1 Monopolistic Competition

There is one firm producing each variety: $\mathcal{M} = 1$, $N = \Omega$. So changes in N coincide with change in Ω. The markup is given by

$$\mu = \frac{\varepsilon}{\varepsilon - 1}$$

B.3.2 Cournot competition

There are $\mathcal{M} >> 1$ firms producing each variety. When $\mathcal{M}$ is large so that firms do not internalize effect on price index P, the markup is

$$\mu = \frac{\varepsilon}{\varepsilon - \frac{1}{\mathcal{M}}}$$

B.3.3 Oligopoly.

There are $\mathcal{M} > 1$ firms producing each variety, but $\mathcal{M}$ small, so that firms do internalize effect on price index P.

With nested CES preferences, the demand elasticity under Bertrand competition is given by

$$\varepsilon = \eta \frac{\mathcal{M} - 1}{\mathcal{M}} + \frac{1}{\mathcal{M}} \sigma$$

where $\eta > \sigma$ is elasticity of substitution across firms producing the same good. This implies a markup

$$\mu = \frac{\eta + \frac{\sigma - \eta}{\mathcal{M}}}{\eta - 1 + \frac{\sigma - \eta}{\mathcal{M}}}$$

which is decreasing in $\mathcal{M}$.

With nested CES preferences, the residual demand elasticity under Cournot competition is given by

$$\varepsilon = \left[\frac{1}{\eta}\left(\frac{\mathcal{M} - 1}{\mathcal{M}}\right) + \frac{1}{\sigma}\frac{1}{\mathcal{M}}\right]^{-1}$$

so that the markup is

$$\mu = \frac{\sigma\eta}{\sigma(\eta-1)+\frac{\sigma-\eta}{\mathcal{M}}}$$

which is also decreasing in $\mathcal{M}$.

C Details of Generalizations of Production Structure

For the sake of simplifying the notation in this Appendix we often abstract from occupations and work directly with

$$L_Y := \prod_{j=1}^{J} L_{jY}^{\eta_{jY}}$$
$$L_N := \prod_{j=1}^{J} L_{jN}^{\eta_{jN}}$$

and wage indices

$$W_Y := \prod_{j=1}^{J} \left(\frac{W_j}{\eta_{jY}}\right)^{\eta_{jY}}$$
$$W_N := \prod_{j=1}^{J} \left(\frac{W_j}{\eta_{jN}}\right)^{\eta_{jN}} .$$

C.1 Variety-specific DRS in production

Upstream Sector Each variety $i \in [0, N]$ in this economy is produced by a variety-specific representative upstream firm. Upstream firm i hires labor l_{Yi} in a competitive labor market at wage W_Y, which it uses to produce an intermediate good M_i. M_i is then sold to retailers at price P_{Ui}. The upstream firm thus chooses labor and output to maximize profits π_{Ui}:

$$\begin{aligned} \pi_{Ui} &:= \max_{L_{Yi},M_i} P_{Ui}M_i - W_Y l_{Yi} \\ &\text{subject to} \\ M_i &= Z_U L_{Yi}^{\gamma_Y} \end{aligned}$$

Downstream Sector A unit measure continuum of identical downstream firms each hire labor L_N in a competitive labor market at wage W_N, which they use to manage product lines. Each product line i generates gross profits Π_i, which the downstream firm's expansion department takes as given when deciding on the number of lines to operate. The firm thus chooses labor and product lines to maximize net profits Π_D:

$$\begin{aligned} \Pi_D &:= \max \int_0^N \Pi_i di - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\gamma_N} \end{aligned}$$

The firm's pricing department for product line i purchases m_i units of good from the upstream firm i, and then sells to consumers at a markup $\mu \geq 1$ over marginal cost P_{Ui}. Hence the price charged for product line i is

$$p_i = \mu P_{Ui} \tag{16}$$

Factor shares We focus on symmetric equilibria in which $p_i = p\ \forall i$, $P_{Ui} = P_U\ \forall i$, $M_i = M\ \forall i$, $m_i = m\ \forall i$ and $y_i = y\ \forall i$. Market clearing for intermediate goods then implies that

$$\begin{aligned}
\Pi_i &= \Pi \\
\pi_{Ui} &= \pi_U \\
\Pi_U &= N\pi_U \\
L_{Yi} &= L_Y \\
Y &= Ny \\
L &= L_Y + L_N \\
L_Y &= Nl_Y
\end{aligned}$$

Nominal GDP in this economy is $pY = pNy = pMy$. The shares of total income accruing to Y-type labor and N-type are defined as

$$S_{LN} := \frac{W_N L_N}{pY} \text{ and } S_{LY} := \frac{W_Y L_Y}{pY},$$

and the overall labor share is defined as $S_L = S_{LN} + S_{LY}$. The overall profit share in the economy is given by the sum of profit shares in both sectors, $S_\Pi = S_U + S_D$, where

$$S_U := \frac{\Pi_U}{pY} \text{ and } S_D := \frac{\Pi_D}{pY}.$$

To obtain expressions for factor shares notice that first order conditions in the wholesale and retail sector are

$$\begin{aligned}
W_Y &= P_U \gamma_Y \frac{M}{l_Y} \\
W_N &= \gamma_N \frac{N}{L_N} y \left(p - P_U\right)
\end{aligned}$$

which in equilibrium (using equation 16) can be rewritten as

$$\begin{aligned}
\frac{W_Y L_Y}{pNy} &= \gamma_Y \frac{1}{\mu} \\
\frac{W_N L_N}{pNy} &= \gamma_N \left(1 - \frac{1}{\mu}\right).
\end{aligned}$$

This shows that expressions for labor share remain unchanged. Lemma 1 still holds. Since the formulas for factor shares do not change Theorems 3, 1 Similarly, 2 is also unaffected.

C.2 Integrated upstreams and downstream sectors as single firms

In this section we study a version of the model in which retailers have to produce goods themselves. Consider a single firm that chooses how much to produce M and how many markets to sell in, N, taking the inverse demand curve in each market as given,

$$\begin{aligned}
\Pi =& \max_{L_Y, L_N, y_i, p_i, N, M} \int_0^N p_i y_i di - W_N L_N - W_Y L_Y \\
& \text{subject to} \\
N =& L_N^{\gamma_N} \\
M =& L_Y^{\gamma_Y} \\
M =& \int_0^N y_i di \\
p_i =& p\left(y_i, \quad \underbrace{Y, N, P}_{\text{taken as given}} \right)
\end{aligned}$$

First order conditions are

$$N: \ p(y, \cdot)\, y_i = \frac{1}{\gamma_N} W_N N^{\frac{1}{\gamma_N}-1} + \frac{1}{\gamma_Y} W_Y y \left(\int_0^N y_i di \right)^{\frac{1}{\gamma_Y}-1}$$

$$y_i: \ p'(y_i, \cdot)\, y_i + p(y_i, \cdot) = \frac{1}{\gamma_Y} W_Y \left(\int_0^N y_i di \right)^{\frac{1}{\gamma_Y}-1}$$

and in a symmetric equilibrium with $y_i = y$ and $p_i = p = P$ they can be rewritten as

$$\begin{aligned}
1 &= \frac{\frac{1}{\gamma_N} W_N N^{\frac{1}{\gamma_N}}}{pyN} + \frac{1}{\gamma_Y} W_Y \frac{(Ny)^{\frac{1}{\gamma_Y}}}{pNy} \\
1 - \varepsilon_{p,y} &= \frac{1}{\gamma_Y} W_Y \frac{(Ny)^{\frac{1}{\gamma_Y}-1}}{pN}
\end{aligned}$$

where $\varepsilon_{p,y} := -\frac{p'(y,\cdot)y}{p}$. Moreover, since $pNy = pY$, we have

$$\begin{aligned}
\frac{W_Y L_Y}{pY} &= \gamma_Y (1 - \varepsilon_{p,y}) \\
\frac{W_N L_N}{pY} &= \gamma_N \varepsilon_{p,y}
\end{aligned}$$

Define $\mu := \frac{1}{1-\varepsilon_{p,y}}$ to get

$$\begin{aligned}
S_{LY} &:= \frac{W_Y L_Y}{pY} = \gamma_Y \frac{1}{\mu} \\
S_{LN} &:= \frac{W_N L_N}{pY} = \gamma_N \left(1 - \frac{1}{\mu} \right).
\end{aligned}$$

The factor shares are therefore as in Section 2.2 and Theorems 1, 2 and 3 still hold.

The key assumption needed for this is that there is a single production function for M. Consider instead an economy in which there are variety-specific production functions (but with a common γ_Y)

$$m_i = \ell_{iY}^{\gamma_Y}$$

The problem of single firm that chooses how much of each variety to produce $\{m_i\}$ and how many markets to sell in, N, taking the inverse demand curve in each market as given is

$$\begin{aligned}
\Pi =& \max_{L_N,\{y_i,p_i,,m_i,\ell_{iY}\},N} \int_0^N \left[p_i y_i - W_Y \ell_{iY}\right] di - W_N L_N \\
& \text{subject to} \\
N =& L_N^{\gamma_N} \\
m_i =& \ell_{iY}^{\gamma_Y} \ \forall_i \\
y_i =& m_i \ \forall_i \\
p_i =& p\left(y_i, \underbrace{Y, N, P}_{\text{taken as given}}\right)
\end{aligned}$$

It is useful to consider the cost minimization problem in each product line (or variety) :

$$\begin{aligned}
TC_i :=& \min_{m_i \ell_{iY}} W_Y \ell_{jY} \\
& \text{subject to} \\
m_i =& \ell_{iY}^{\gamma_Y}
\end{aligned}$$

which results in the expression for marginal cost

$$MC_i = \frac{1}{\gamma_Y} W_Y m_i^{\frac{1}{\gamma_Y}-1}$$

First order conditions are

$$\begin{aligned}
N:\ p(y,\cdot)\, y_i =& \frac{1}{\gamma_N} W_N N^{\frac{1}{\gamma_N}-1} + TC_i(y_i) \\
y_i:\ p'(y_i,\cdot)\, y_i + p(y_i,\cdot) =& MC_i(y_i)
\end{aligned}$$

and in a symmetric equilibrium with $y_i = y$ and $p_i = p = P$ they can be rewritten as

$$\begin{aligned}
1 =& \frac{1}{\gamma_N} \frac{W_N N^{\frac{1}{\gamma_N}}}{pyN} + \frac{W_Y y^{\frac{1}{\gamma_Y}} N}{pyN} \\
1 - \varepsilon_{p,y} =& \frac{1}{\gamma_Y} \frac{W_Y y^{\frac{1}{\gamma_Y}} N}{pyN}
\end{aligned}$$

where $\varepsilon_{p,y} := -\frac{p'(y,\cdot)y}{p}$. Moreover, since $pNy = pY$, we have

$$\begin{aligned}
\frac{W_Y L_Y}{pY} =& \gamma_Y \left(1 - \varepsilon_{p,y}\right) \\
\frac{W_N L_N}{pY} =& \gamma_N \left(1 - \gamma_Y \left(1 - \varepsilon_{p,y}\right)\right)
\end{aligned}$$

Define $\mu := \frac{1}{1-\varepsilon_{p,y}}$ to get

$$S_{LY} := \frac{W_Y L_Y}{pY} = \gamma_Y \frac{1}{\mu}$$
$$S_{LN} := \frac{W_N L_N}{pY} = \gamma_N \left(1 - \gamma_Y \frac{1}{\mu}\right).$$

This shows that the expression for S_{LN} is different now. The reason is that a part of what we called upstream profits in Section 2.1 remunerates N-type labor. In consequence, Theorem 3 does not hold since

$$S_L = \gamma_Y (1 - \gamma_N) \frac{1}{\mu} + \gamma_N$$

and an increase in the markup always decreases the labor share. Theorem 1 is still valid, but S_{LN} is less sensitive to changes in the markup - while its "downstream" component stemming from monopolistic rents, $\gamma_N \left(1 - \frac{1}{\mu}\right)$, is positively related to increases in the markup, the component resulting from decreasing returns to scale in production, $\gamma_N \frac{1}{\mu} (1 - \gamma_Y)$, is negatively related.

C.3 Capital and other factors of productions

In this section we allow for multiple factors of productions. We show that adding capital and other factors of production has no impact on our results if production functions in the downstream and upstream are multiplicative in labor and some arbitrary aggregators of other factors of productions. Let

$$M = Z_U L_Y^{\gamma_Y} f_Y (\{X_Y\})$$

and

$$N = Z_D L_N^{\gamma_N} f_N (\{X_N\})$$

where $\{X_Y\} = \{X_{1Y}, X_{2Y}, X_{3Y}, \ldots\}$ and $\{X_N\} = \{X_{1N}, X_{2N}, X_{3N}, \ldots\}$ are *factors of production* (i.e. they are not intermediate inputs) excluding labor. Assume $0 \leq \gamma_Y \leq 1$ and $0 \leq \gamma_N \leq 1$. Let f_Y, f_N be arbitrary nondecreasing function. We need to assume $L_Y^{\gamma_Y} f_Y (\{X_Y\})$ is concave in $(L_Y, \{X_Y\})$ and $L_N^{\gamma_N} f_N (\{X_N\})$ is concave in $(L_N, \{X_N\})$. Moreover, let $L = L_N + L_Y$ so that there are no other uses of labor in this economy. Since

$$\frac{\partial M}{\partial L_Y} = \gamma_Y \frac{M}{L_Y}$$
$$\frac{\partial N}{\partial L_N} = \gamma_N \frac{N}{L_N}$$

and $Y = Ny = M$ (because we assumed $\{X_Y\}$ and $\{X_N\}$ are not intermediate inputs) we still have

$$W_Y L_Y = \frac{1}{\mu} \gamma_Y pY$$
$$W_N L_N = \left(1 - \frac{1}{\mu}\right) \gamma_N pY$$

and Theorems 1 and 2 as well as the part of Theorem 3 concerning the behavior of the labor share still hold.

C.4 CES Production Function

We explore the effects of extending the production functions in each sector to Constant Elasticity of Substitution functions:

$$M = Z_U \left[b_Y L_Y^{\rho_Y} + (1-b_Y) X_Y^{\rho_Y}\right]^{\frac{1}{\rho_Y}\theta_Y}$$
$$N = Z_D \left[b_N L_N^{\rho_N} + (1-b_N) X_N^{\rho_N}\right]^{\frac{1}{\rho_N}\theta_N}$$

where (X_Y, X_N) are some other factors of productions (or aggregates of other factors of production). We assume $b_Y, b_N, \theta_Y, \theta_N \in (0,1)$ and $\rho_Y, \rho_N \leq 1$. With an exception of different production functions, we keep problems faced by the upstream and the downstream firms unchanged. We show the conditions under which Theorems 1 and 3 hold.

Lemma 2. *In an economy with this production structure, the equilibrium factor shares are given by*

$$S_{LY} = \frac{1}{\mu}\gamma_Y(t_Y)$$
$$S_{LN} = \left(1-\frac{1}{\mu}\right)\gamma_N(t_N)$$
$$S_L = \frac{1}{\mu}\gamma_Y(t_Y) + \left(1-\frac{1}{\mu}\right)\gamma_N(t_N)$$

where

$$\gamma_Y(t) := \theta_Y \frac{b_Y t^{\rho_Y}}{b_Y t^{\rho_Y} + (1-b_Y)}$$
$$\gamma_N(t) := \theta_N \frac{b_N t^{\rho_N}}{b_N t^{\rho_N} + (1-b_N)}$$

and

$$t_Y := \frac{L_Y}{X_Y}$$
$$t_N := \frac{L_N}{X_N}$$

Proof. First order conditions with respect to Y- and N-types of labor are, in a symmetric equilibrium with $\Pi_i = \Pi$,

$$W_Y = P_U M \theta_Y \frac{b_Y L_Y^{\rho_Y - 1}}{b_Y L_Y^{\rho_Y} + (1-b_Y) X_Y^{\rho_Y}}$$
$$W_N = \Pi N \theta_N \frac{b_N L_N^{\rho_N - 1}}{b_N L_N^{\rho_N} + (1-b_N) X_N^{\rho_N}}$$

Now use the market clearing $M = Nm = Ny = Y$ together with $\Pi = (p - P_U)\,y$ and $p = \mu P_U$ to obtain

$$
\begin{aligned}
S_{LY} &= \frac{1}{\mu}\underbrace{\theta_Y \frac{b_Y L_Y^{\rho_Y - 1}}{b_Y L_Y^{\rho_Y} + (1-b_Y)\,X_Y^{\rho_Y}}}_{\equiv \gamma_Y\left(\frac{L_Y}{X_Y}\right)} \\
S_{LN} &= \left(1 - \frac{1}{\mu}\right)\underbrace{\theta_N \frac{b_N L_N^{\rho_N - 1}}{b_N L_N^{\rho_N} + (1-b_N)\,X_N^{\rho_N}}}_{\equiv \gamma_N\left(\frac{L_N}{X_N}\right)} \\
S_L &= \frac{1}{\mu}\gamma_Y\left(\frac{L_Y}{X_Y}\right) + \left(1 - \frac{1}{\mu}\right)\gamma_N\left(\frac{L_N}{X_N}\right).
\end{aligned}
$$

□

Theorem 2 generalizes Theorem 1 to CES production functions.

Theorem 4. *Let*

$$
\begin{aligned}
\xi_{t_Y,\frac{1}{\mu}} &:= \frac{1}{t_Y}\frac{dt_Y}{d\frac{1}{\mu}}\frac{1}{\mu} \\
\xi_{t_N,\frac{1}{\mu}} &:= \frac{1}{t_N}\frac{dt_N}{d\frac{1}{\mu}}\frac{1}{\mu}.
\end{aligned}
$$

If $\rho_Y \xi_{t_Y,\frac{1}{\mu}} \geq 0$ and $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$ then an increase in the markup μ leads to a decrease in the income share of Y-type labor and an increase in the income share of N-type labor.

Proof. In this economy

$$
\begin{aligned}
\frac{dS_{LY}}{d\frac{1}{\mu}} &= \gamma_Y\left[1 + \frac{(1-b_Y)}{b_Y t_Y^{\rho_Y} + (1-b_Y)}\rho_Y \xi_{t_Y,\frac{1}{\mu}}\right] \\
\frac{dS_{LN}}{d\frac{1}{\mu}} &= -\gamma_N\left[1 - (\mu - 1)\frac{(1-b_N)}{b_N t_N^{\rho_N} + (1-b_N)}\rho_N \xi_{t_N,\frac{1}{\mu}}\right]
\end{aligned}
$$

and $\frac{(1-b_Y)}{b_Y t_Y^{\rho_Y} + (1-b_Y)} > 0$ and $(\mu - 1)\frac{(1-b_N)}{b_N t_N^{\rho_N} + (1-b_N)} > 0$. If $\rho_Y \xi_{t_Y,\frac{1}{\mu}} \geq 0$ then $\frac{dS_{LY}}{d\frac{1}{\mu}} > 0$. If $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$ then $\frac{dS_{LN}}{d\frac{1}{\mu}} < 0$. Therefore under those conditions we have

$$
\begin{aligned}
\frac{dS_{LY}}{d\mu} &< 0 \\
\frac{dS_{LN}}{d\mu} &> 0.
\end{aligned}
$$

□

Theorem 2 shows that it is not always the case that an increase in the markup increases the share of income going to N-type labor and reduces the share of income accruing to Y-type labor. Redistribution depends on elasticities of substitution between labor and other factors of production and on the effect of changes in the markup on the ratio of labor to other inputs. For an increase in the markup to decrease S_{LY} it is sufficient that the ratio of Y-type labor to other factors of production goes down ($\xi_{t_Y,\frac{1}{\mu}} \geq 0$) when labor and other factors of production are substitutes ($\rho_Y \geq 0$) and increases ($\xi_{t_Y,\frac{1}{\mu}} \leq 0$) when they are complements ($\rho_Y \leq 0$). An increase in the markup will increase S_{LN} when the ratio of N-type labor to other inputs increases ($\xi_{t_N,\frac{1}{\mu}} \leq 0$) when N-type labor and other inputs are complements ($\rho_N \leq 0$) and decreases ($\xi_{t_N,\frac{1}{\mu}} \geq 0$) if they are substitutes ($\rho_N \geq 0$). These conditions are sufficient so it could be the case that, for example, $\frac{dS_{LY}}{d\mu} \leq 0$ even if $\rho_Y \xi_{t_Y,\frac{1}{\mu}} < 0$. This happens as long as $1 + \rho_Y \xi_{t_Y,\frac{1}{\mu}} > -\frac{b_Y}{1-b_Y} t_Y^{\rho_Y}$. The advantage of providing sufficient conditions in terms of $\rho_Y \xi_{t_Y,\frac{1}{\mu}}$ and $\rho_N \xi_{t_N,\frac{1}{\mu}}$ is that these depend only on the product of elasticity of substitution and the direction of change in the L/X ratio in response to a change in the markup. As an example suppose that $\rho_Y = \rho_N = \rho > 0$ so that labor and other factors of production are substitutes in both sectors. An increase in the aggregate markup will redistribute income from Y- to N- type labor when it decreases the ratio L/X in the upstream sector and increases it in the downstream sector. Note that $\xi_{t_Y,\frac{1}{\mu}}, \xi_{t_N,\frac{1}{\mu}}, \gamma_Y(t_Y), \gamma_N(t_N)$ are equilibrium objects so in principle they could depend on the level of markup and other variables.

In contrast to the baseline model with Cobb-Douglas technology, S_{LN} and S_{LY} might both go up or down after an increase in the markup. Even if $\rho_Y \xi_{t_Y,\frac{1}{\mu}} \geq 0$, $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$ and $\gamma_N(t_N) > \gamma_Y(t_Y)$ it is still possible that $\frac{dS_L}{d\mu} < 0$. For example, S_{LY} can fall by so much that S_L drops despite an increase in S_{LN}. However, in a special case in which the upstream sector uses a Cobb-Douglas technology ($\rho_Y = 0$) and $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$, if $\gamma_N(t_N) > \gamma_Y(t_Y)$, an increase in the markup will increase the labor share S_L.

Theorem 5. *Let $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$ and $\rho_Y = 0$. If $\gamma_Y(t_Y) < \gamma_N(t_N)$ then an increase in the markup μ leads to a decrease in the income share of Y-type labor and an increase in the income share of N-type labor.*

Proof. We have

$$\frac{dS_L}{d\frac{1}{\mu}} = \gamma_Y \left[1 + \frac{(1-b_Y)}{b_Y t_Y^{\rho_Y} + (1-b_Y)} \rho_Y \xi_{t_Y,\frac{1}{\mu}}\right] - \gamma_N \left[1 - (\mu - 1) \frac{(1-b_N)}{b_N t_N^{\rho_N} + (1-b_N)} \rho_N \xi_{t_N,\frac{1}{\mu}}\right].$$

If $\gamma_Y(t_Y) < \gamma_N(t_N)$, $\rho_N \xi_{t_N,\frac{1}{\mu}} \leq 0$ and $\rho_Y = 0$

$$\begin{aligned}\frac{dS_L}{d\frac{1}{\mu}} &= \gamma_Y - \gamma_N \left[1 - (\mu - 1) \frac{(1-b_N)}{b_N t_N^{\rho_N} + (1-b_N)} \rho_N \xi_{t_N,\frac{1}{\mu}}\right] \\ &\geq \gamma_Y - \gamma_N\end{aligned}$$

□

C.5 Entry in the upstream sector

We generalize the model to allow entry in the upstream sector. The upstream firm operates E plants. For each plant the problem is

$$\begin{gathered}\Pi_U := \max_{L_Y, M} P_U M - W_Y L_Y \\ \text{subject to} \\ M = Z_U L_Y^{\gamma_Y}\end{gathered}$$

and the first order condition with respect to L_Y is the same as in the baseline model (see equation 14). To operate plants the upstream firm needs to hire N-type labor. The problem is

$$\begin{gathered}\max_{L_{NU}, E} E\Pi_U - W_N L_{NU} \\ \text{subject to} \\ E = Z_E L_{NU}\end{gathered}$$

with the first order condition

$$W_N = \frac{E}{L_{NU}} \Pi_U$$

The problem of the downstream sector remains unchanged. Market clearing is

$$yN = EM$$

and in a symmetric equilibrium factor shares are

$$\begin{aligned} S_{LY} &:= \frac{EW_Y L_Y}{pEM} \\ S_{LN} &:= \frac{W_N \left(L_N + L_{NU}\right)}{pEM} \end{aligned}$$

i.e.

$$\begin{aligned} S_{LN} &= \left(1 - \frac{1}{\mu}\right) \gamma_N + \frac{1}{\mu} \left(1 - \gamma_Y\right) \\ S_{LY} &= \frac{1}{\mu} \gamma_Y \end{aligned}$$

and the overall labor share is

$$S_L = \left(1 - \frac{1}{\mu}\right) \gamma_N + \frac{1}{\mu}.$$

Theorem 3 does not hold. In this case an increase in the markup always leads to a fall in the labor share. Theorem 1 holds only if

$$\gamma_N > 1 - \gamma_Y.$$

While the labor share of the Y-type labor labor always falls when the markup increases, the effect on S_{LN} is ambiguous because some N-type labor is used in the upstream sector. For Theorem 1 to hold it has to be the case that the incentive to hire N-type labor in the upstream sector is weak. This happens when the profit share in each plant's revenue is low, i.e. when γ_Y is sufficiently large. However, an increase in the markup always increases the share of N-type labor income in the aggregate labor income, $\frac{S_{LN}}{S_L}$.

$$\begin{aligned} \frac{d \log \left(\frac{S_{LN}}{S_L}\right)}{d \frac{1}{\mu}} &= \frac{-\gamma_N + 1 - \gamma_Y}{\left(1 - \frac{1}{\mu}\right) \gamma_N + \frac{1}{\mu} \left(1 - \gamma_Y\right)} - \frac{1 - \gamma_N}{\left(1 - \frac{1}{\mu}\right) \gamma_N + \frac{1}{\mu}} \\ &< 0 \iff \gamma_N > 0. \end{aligned}$$

C.6 Markups In Labor Market and Upstream Sector

We generalize the model to allow for markups in the upstream sector and labor markets. We assume that the upstream firm is a monopsonist in the labor market and a monopolist in the product market. First order condition of the firm is

$$\frac{\mu_U}{\mu_{L_Y}} W_Y = P_U \gamma_Y \frac{M}{L_Y} \tag{17}$$

where $\mu_U \geq 1$ and $\mu_{L_Y} \leq 1$. Similarly, we assume some degree of monopsonistic power ($\mu_{L_N} \leq 1$) in the downstream sector and so

$$\frac{1}{\mu_{L_N}} \frac{W_N}{p} = \gamma_N \frac{N}{L_N} \Pi \tag{18}$$

and in symmetric equilibrium factor shares become

$$S_{LY} = \frac{\mu_{L_Y}}{\mu_U} \frac{1}{\mu_D} \gamma_Y$$
$$S_{LN} = \mu_{L_N} \left(1 - \frac{1}{\mu_D}\right) \theta \gamma_N$$

where μ_{L_Y} and μ_{L_N} are markups in the labor markets for Y-type labor and N-type labor, and μ_D is the markup in the downstream sector (on which we focus in this paper).

We assume that markups are independent of each other and exogenous. The presence of markups in these other markets does not change Theorem 1: an increase in the markup still redistributes factor income away from Y-type labor and toward N-type labor. However, the condition for Theorem 3 is modified. Positive co-movement between the (downstream) markup and the labor share requires $\gamma_N > \frac{\mu_{L_Y}}{\mu_{L_N} \mu_U} \gamma_Y$. When $\mu_{L_N} = \mu_{L_Y}$, the presence of a markup in the upstream sector ($\mu_U > 1$) thus expands the set of (γ_N, γ_Y) which are consistent with co-movement observed in US data. We also have

$$\frac{\partial S_{LY}}{\partial \mu_U} < 0$$
$$\frac{\partial S_{LN}}{\partial \mu_U} = 0$$
$$\frac{\partial S_L}{\partial \mu_U} < 0$$

meaning that an increase in upstream markup reduces the Y-type share and the overall labor share. It has no effect on the N-type share. In addition

$$\frac{\partial S_{LY}}{\partial \mu_{L_Y}} > 0$$
$$\frac{\partial S_{LN}}{\partial \mu_{L_Y}} = 0$$
$$\frac{\partial S_L}{\partial \mu_{L_Y}} > 0$$

and

$$\frac{\partial S_{LY}}{\partial \mu_{L_N}} = 0$$
$$\frac{\partial S_{LN}}{\partial \mu_{L_N}} > 0$$
$$\frac{\partial S_L}{\partial \mu_{L_N}} > 0$$

so a *decrease* in the degree of monoposonistic power always increases the overall labor share in the economy and that happens through an increase in the share of one type of labor. Any type of markup or markdown that appears as a wedge between the real wage and the marginal rate of substitution of households (as it is often the case in New Keynesian models with wage rigidities) would not affect our results as long as μ_{L_Y} and μ_{L_N} would not be affected by it.

So far we have simply assumed the presence of exogenous wedges $\mu_U, \mu_{L_Y}, \mu_{L_N}$ in equations 17 and 18. Below we show an example in which these wedges are functions of structural parameters of the model. Suppose there is a representative household with preferences

$$\log C-\chi_Y\frac{L_Y^{1+\frac{1}{\varphi_Y}}}{1+\frac{1}{\varphi_Y}}-\chi_N\frac{L_N^{1+\frac{1}{\varphi_N}}}{1+\frac{1}{\varphi_N}}$$

where $C=\mathcal{C}\left(\{c_\omega\}_{\omega\in[0,\Omega]},\Omega\right)$ is a symmetric homothetic aggregator over distinct varieties c_ω and Ω is the measure of varieties. We assume this aggregator features no love of variety. The representative household solves

$$\max_{C,L_Y,L_N}\log C-\chi_Y\frac{L_Y^{1+\frac{1}{\varphi_Y}}}{1+\frac{1}{\varphi_Y}}-\chi_N\frac{L_N^{1+\frac{1}{\varphi_N}}}{1+\frac{1}{\varphi_N}}$$
$$\text{subject to}$$
$$pC=W_YL_Y+W_NL_N+\Pi_U+\Pi_D$$

First order conditions are

$$\chi_YCL_Y^{\frac{1}{\varphi_Y}}=W_Y$$
$$\chi_NCL_N^{\frac{1}{\varphi_N}}=W_N$$

Below we describe an example of micro-founded environment in which markup variation arises as a result of exogenous variation in a structural parameter. There is an upstream firm which takes the household's labor supply schedule as given and solves

$$\Pi_U:=\max_{L_Y,U}(1-\tau)P_UM-W_YL_Y$$
$$\text{subject to}$$
$$M=Z_UL_Y^{\gamma_Y}$$
$$L_Y=\left(\frac{W_Y}{\chi_YC}\right)^{\varphi_Y}$$

where τ is a tax rate on the upstream firm's revenue.[15] The first order condition is

$$\left(1+\frac{1}{\varphi_Y}\right)W_Y=(1-\tau)P_U\gamma_Y\frac{M}{L_Y}.$$

The downstream firm also takes the household's labor supply schedule as given and solves

$$\Pi_D:=\max_{L_N,N}\int_0^N\Pi_i di-W_NL_N$$
$$\text{subject to}$$
$$N=Z_DL_N^{\gamma_N}$$
$$L_N=\left(\frac{W_N}{\chi_NC}\right)^{\varphi_N}$$

and first order condition with respect to L_N is (in a symmetric equilibrium with $\Pi_i=\Pi$ for all i)

$$\left(1+\frac{1}{\varphi_N}\right)W_N=\gamma_NZ_DL_N^{\gamma_N-1}\Pi.$$

[15] Alternatively we could assume that there is a continuum of monopolistically competitive wholesalers and the retailer's pricing department has to purchase a CES bundle of them.

Define

$$\mu_{L_Y} := \left(1 + \frac{1}{\varphi_Y}\right)^{-1}$$
$$\mu_U := (1 - \tau)^{-1}$$
$$\mu_{L_N} := \left(1 + \frac{1}{\varphi_N}\right)^{-1}$$

to obtain equations 17 and 18. In this framework shifts in μ_{L_Y} and μ_{L_N} can be interpreted as changes in Frisch elasticities φ_Y, φ_N while shifts in μ_U result from changes in the tax rate τ.

C.7 Further processing in the downstream sector

In an economy with further processing in the downstream sector the problem of the upstream firm is as in Section 2.1:

$$\Pi_U := \max_{M, \{L_{jY}\}_{j=1}^J} P_U M - \sum_{j=1}^J W_j L_{jY}$$
$$\text{subject to}$$
$$M = Z_U \left(\prod_{j=1}^J L_{jY}^{\eta_{jY}}\right)^{\gamma_Y}.$$

Similarly, the problem of the downstream firm's expansion department is as in Section 2.1:

$$\Pi_D := \max_{N, \{L_{jN}\}_{j=1}^J} \int_0^N \Pi_i di - \sum_{j=1}^J W_j L_{jN}$$
$$\text{subject to}$$
$$N = Z_D \left(\prod_{j=1}^J L_{jN}^{\eta_{jN}}\right)^{\gamma_N}$$

The difference is that now each product line $i \in [0, N]$ purchases the quantity m_i of the upstream good at price P_U and combines it with Y-type labor to produce y_i, where

$$y_i = m_i^{1-\gamma_D} \left(\prod_{j=1}^J \ell_{jiYD}^{\eta_{jY}}\right)^{\gamma_D}$$

so the gross profits in each product line are given by

$$\Pi_i := p_i y_i - P_U m_i - \sum_{j=1}^J W_j \ell_{jiYD}.$$

Aggregate output of the downstream sector is

$$Y := \int_0^N y_i di.$$

We focus on symmetric equilibria in which $p_i = p \ \forall i$, $m_i = m \ \forall i$ and $y_i = y \ \forall j$. Market clearing for intermediate goods then implies that

$$mN = M$$

and for each occupation j we have

$$L_j = L_{jY} + L_{jN} + N\ell_{jYD}.$$

Nominal GDP is defined as pY. Here, however, $y \neq m$ so it is not longer the case that $pY = pM$. The shares of total income accruing to Y-type labor and N-type labor are defined as

$$S_{LN} := \frac{\sum_{j=1}^{J} W_j L_{jN}}{pY} \text{ and } S_{LY} := \frac{\sum_{j=1}^{J} W_j \left(L_{jN} + N\ell_{jYD}\right)}{pY},$$

and the overall labor share is defined as $S_L = S_{LN} + S_{LY}$. The overall profit share in the economy is given by the sum of profit shares in the upstream and downstream sectors, $S_\Pi = S_U + S_D$, where

$$S_U := \frac{\Pi_U}{pY} \text{ and } S_D := \frac{\Pi_D}{pY}.$$

The share of total income accruing to occupation j is defined as

$$S_j := \frac{W_j L_j}{pY}.$$

Lemma 3. *In an economy with this production structure, the equilibrium factor shares are given by*

$$\begin{aligned}
S_L &= \frac{1}{\mu}\left(\gamma_D + (1-\gamma_D)\gamma_Y\right) + \left(1 - \frac{1}{\mu}\right)\gamma_N \\
S_{LY} &= \frac{1}{\mu}\left(\gamma_D + (1-\gamma_D)\gamma_Y\right) \\
S_{LN} &= \left(1 - \frac{1}{\mu}\right)\gamma_N \\
S_U &= \frac{1}{\mu}(1-\gamma_D)(1-\gamma_Y) \\
S_D &= \left(1 - \frac{1}{\mu}\right)(1-\gamma_N) \\
S_j &= \frac{1}{\mu}\eta_{jY}\left(\gamma_D + (1-\gamma_D)\gamma_Y\right) + \left(1 - \frac{1}{\mu}\right)\eta_{jN}\gamma_N
\end{aligned}$$

and the inverse markup is related to the ratio of price indices $\varrho := \frac{p}{P_U}$ as

$$\frac{1}{\mu} = \left(\frac{1}{\varrho}\right)^{1-\gamma_D}\left(\frac{W_Y/p}{\gamma_Y}\right)^{\gamma_D}\left(\frac{1}{1-\gamma_D}\right)^{1-\gamma_D}.$$

Proof. In this case it is easier to begin with the downstream sector. The downstream firm solves

$$\Pi_D := \max_{N,\{L_{jN}\}_{j=1}^{J}} \int_0^N \Pi_i di - \sum_{j=1}^{J} W_j L_{jN}$$

subject to

$$N = Z_D\left(\prod_{j=1}^{J} L_{jN}^{\eta_{jN}}\right)^{\gamma_N}$$

which, in a symmetric equilibrium with $\Pi_i = \Pi$ for all i, can be written as

$$\Pi_D := \max_{N, \{L_{jN}\}_{j=1}^{N}} N\Pi - \sum_{j=1}^{J} W_j L_{jN}$$
$$\text{subject to}$$
$$N = Z_D \left(\prod_{j=1}^{J} L_{jN}^{\eta_{jN}} \right)^{\gamma_N}$$

and the first order condition with respect toL_{kN} is

$$W_j = \gamma_N \eta_{kN} \frac{N}{L_{kN}} \Pi. \tag{19}$$

The problem of each product line is

$$\Pi_i := \max_{y_i, m_i, \{\ell_{jiYD}\}_{j=1}^{J}} p_i y_i - P_U m_i - \sum_{j=1}^{J} W_j \ell_{jiYD}$$
$$\text{subject to}$$
$$y_i = m_i^{1-\gamma_D} \left(\prod_{j=1}^{J} \ell_{jiYD}^{\eta_{jY}} \right)^{\gamma_D}.$$

so the gross profits in each product line are given by

$$\Pi_i := p_i y_i - P_U m_i - \sum_{j=1}^{J} W_j \ell_{jiYD}.$$

Total cost of each product line is

$$TC_i(y_i) := y_i \left(\frac{P_U}{1-\gamma_D} \right)^{1-\gamma_D} \left[\frac{1}{\gamma_D} \prod_{j=1}^{J} \left(\frac{W_j}{\eta_{jY}} \right)^{\eta_{jY}} \right]^{\gamma_D}$$

so the gross profits are

$$\Pi_i := \left[p_i - \left(\frac{P_U}{1-\gamma_D} \right)^{1-\gamma_D} \left[\frac{1}{\gamma_D} \prod_{j=1}^{J} \left(\frac{W_j}{\eta_{jY}} \right)^{\eta_{jY}} \right]^{\gamma_D} \right] y_i.$$

We also have a first order condition with respect to ℓ_{kiYD}:

$$W_k = p_i \gamma_D \eta_{jY} \frac{y_i}{\ell_{kiYD}}$$

We assume price is set as

$$p_i = \mu MC_i(y_i)$$

where $MC_i(y_i) := \frac{dTC_i(y_i)}{dy_i}$. Define $\varrho := \frac{p}{P_U}$. We then have (in a symmetric equilibrium) obtain equation

(5)

$$\frac{1}{\mu}=\left(\frac{1}{1-\gamma_D}\frac{1}{\varrho}\right)^{1-\gamma_D}\left[\frac{1}{\gamma_D}\prod_{j=1}^{J}\left(\frac{W_j/p}{\eta_{jY}}\right)^{\eta_{jY}}\right]^{\gamma_D}$$

where

$$\frac{W_Y}{p}:=\prod_{j=1}^{J}\left(\frac{W_j/p}{\eta_{jY}}\right)^{\eta_{jY}}.$$

Notice also that

$$P_U m_i=(1-\gamma_D)\,TC_i\left(y_i\right) \tag{20}$$

$$\sum_{j=1}^{J}W_j\ell_{jiYD}=\gamma_D TC_i\left(y_i\right) \tag{21}$$

In a symmetric equilibrium it is still the case that

$$\Pi=py\left(1-\frac{1}{\mu}\right)$$

so equation (19) can be rewritten as

$$W_k=\gamma_N\eta_{kN}\frac{N}{L_{kN}}py\left(1-\frac{1}{\mu}\right)$$

where we used the fact that $Y=Ny$. It can be then rearranged as

$$\frac{W_kL_{kN}}{pY}=\gamma_N\eta_{kN}\left(1-\frac{1}{\mu}\right).$$

This shows that

$$\begin{aligned}S_{LN}&=\frac{\sum_{j=1}^{J}W_jL_{jN}}{pY}\\&=\gamma_N\left(1-\frac{1}{\mu}\right)\sum_{j=1}^{J}\eta_{jN}\\&=\gamma_N\left(1-\frac{1}{\mu}\right)\end{aligned}$$

and

$$\begin{aligned}S_D&=\frac{\Pi_D}{pY}\\&=\frac{pNy\left(1-\frac{1}{\mu}\right)-\sum_{j=1}^{J}W_jL_{jN}}{pY}\\&=\left(1-\frac{1}{\mu}\right)-\gamma_N\left(1-\frac{1}{\mu}\right)\\&=(1-\gamma_N)\left(1-\frac{1}{\mu}\right).\end{aligned}$$

The upstream firm solves

$$\Pi_U := \max_{M,\{L_{jY}\}_{j=1}^J} P_U M - \sum_{j=1}^J W_j L_{jY}$$

$$\text{subject to}$$

$$M = Z_U \left(\prod_{j=1}^J L_{jY}^{\eta_{jY}}\right)^{\gamma_Y}$$

and the first order condition with respect to L_{kY} is

$$W_k = P_U \gamma_Y \eta_{kY} Z_U \left(\prod_{j=1}^J L_{jY}^{\eta_{jY}}\right)^{\gamma_Y - 1} \frac{\prod_{j=1}^J L_{jY}^{\eta_{jY}}}{L_{kY}}.$$

Use $M = Z_U \left(\prod_{j=1}^J L_{jY}^{\eta_{jY}}\right)^{\gamma_Y}$ to write it as

$$W_k = P_U \gamma_Y \eta_{kY} \frac{M}{L_{kY}}$$

In this case it is not true that $p = \mu P_U$. However, we can use equation (20) to rewrite it as

$$W_k = \gamma_Y \eta_{kY} \frac{(1-\gamma_D)\, y_i \left(\frac{P_U}{1-\gamma_D}\right)^{1-\gamma_D} \left[\frac{1}{\gamma_D} \prod_{j=1}^J \left(\frac{W_j}{\eta_{jY}}\right)^{\eta_{jY}}\right]^{\gamma_D}}{L_{kY}} \tag{22}$$

and use

$$p = \mu \left(\frac{P_U}{1-\gamma_D}\right)^{1-\gamma_D} \left[\frac{1}{\gamma_D} \prod_{j=1}^J \left(\frac{W_j}{\eta_{jY}}\right)^{\eta_{jY}}\right]^{\gamma_D}$$

to get

$$\frac{W_k L_{kY}}{pyN} = \frac{1}{\mu} \gamma_Y \eta_{kY} (1-\gamma_D).$$

Next, to get the share of Y-type labor:

$$\begin{aligned} S_{LY} &= \frac{\sum_{j=1}^J W_j (L_{jY} + N\ell_{jYD})}{pyN} \\ &= \sum_{j=1}^J \frac{1}{\mu} \gamma_Y \eta_{jY} (1-\gamma_D) + \sum_{j=1}^J \frac{1}{\mu} \eta_{jY} \gamma_D \\ &= \frac{1}{\mu} \left[\gamma_Y (1-\gamma_D) + \gamma_D\right] \end{aligned}$$

Finally, upstream profit share is

$$
\begin{aligned}
S_U &:= \frac{\Pi_U}{pY} \\
&= \frac{P_U M - \sum_{j=1}^{J} W_j L_{jY}}{pY} \\
&= \frac{P_U M}{pY}\left(1 - \frac{\sum_{j=1}^{J} W_j L_{jY}}{P_U M}\right) \\
&= (1-\gamma_D)\,\frac{1}{\mu}\,(1-\gamma_Y)
\end{aligned}
$$

which concludes the proof. □

Given Lemma 3 it is straightforward to see that Theorems 1 and 2 still hold. Theorem 3 has to be modified slightly. We have

$$
\frac{\partial S_L}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma_D + (1-\gamma_D)\,\gamma_Y \gtreqless \gamma_N
$$

and

$$
\frac{\partial S_\Pi}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma_N \gtreqless \gamma_D + (1-\gamma_D)\,\gamma_Y.
$$

D Industry Heterogeneity

We assume that there are G different industries, indexed by g, and that final goods are a Cobb-Douglas aggregator of the value-added of each industry,

$$
Y = \prod_{g=1}^{G} Y_g^{\varsigma_g}
$$

where $\varsigma_g \in [0,1]$ are the value added-shares of each industry and satisfy $\sum_{g=1}^{G} \varsigma_g = 1$. Maximization problem of the representative final goods producer is

$$
\max_{Y,\{Y_g\}_{g=1}^{G}} pY - \sum_{g=1}^{G} p_g Y_g
$$

subject to

$$
Y = \prod_{g=1}^{G} Y_g^{\varsigma_g}.
$$

First order conditions of the final goods producer are

$$
Y_g = \varsigma_g \frac{p}{p_g} Y.
$$

Nominal GDP in this economy, pY, is equal to $\sum_{g=1}^{G} p_g Y_g$.

Each industry g has its own representative upstream firm. It takes the price of upstream goods P_{Ug} and the nominal wage in each occupation W_j as given and maximizes profits

$$\max_{\{L_{jYg}\}_{j=1}^{J}} P_{Ug} M_g - \sum_{j=1}^{J} W_j L_{jg,Y}$$

where

$$M_g = Z_{Ug} \left(\prod_{j=1}^{J} L_{jg,Y}^{\eta_{jgY}} \right)^{\gamma_{Yg}}.$$

Labor elasticitiesγ_{Yg} are between 0 and 1 and the industry-specific occupation weights satisfy $\sum_{j=1}^{J} \eta_{jgY} = \sum_{j=1}^{J} \eta_{jgN} = 1 \ \forall g$. Profit maximization implies

$$P_{Ug}\gamma_{Yg} \frac{M_g}{L_{jg,Y}} \times \eta_{jgY} \frac{L_{g,Y}}{L_{jg,Y}} = W_j.$$

where $L_{g,Y} := \prod_{j=1}^{J} L_{jg,Y}^{\eta_{jgY}}$.

In each industry there is a a unit measure continuum of identical downstream firms. They hire N-type labor to manage product lines. Each product line $i_g \in [0, N_g]$ generates gross profits Π_{i_g}, which the downstream firm's expansion department takes as given when deciding on the number of lines to operate. Downstream firms thus choose labor and product lines to maximize net profits Π_{Dg}:

$$\Pi_{Dg} := \max \int_0^{N_g} \Pi_{i_g} di_g - \sum_{j=1}^{J} W_j L_{jNg}$$

subject to

$$N = Z_{Dg} \left(\prod_{j=1}^{J} L_{jg,N}^{\eta_{jgN}} \right)^{\gamma_{Yg}}$$

Profit maximization implies

$$\gamma_{Ng} \frac{N_g}{L_{N,g}} \Pi_g \times \eta_{jgN} \frac{L_{Ng}}{L_{jg,N}} = W_j$$

where $L_{N.g} := \prod_{j=1}^{J} L_{jg,N}^{\eta_{jgN}}$and where we used that in a symmetric equilibrium $\Pi_{i_g} = \Pi_g$.

The firm's pricing department for product line i_g purchases m_{i_g} units of good from the upstream firm in industry g, costlessly differentiates it and then sells to consumers as a differentiated good y_{i_g} at a markup $\mu_g \geq 1$ over marginal cost P_{Ug}. Hence the price charged by product line i_g is

$$p_{i_g} = \mu_g P_{Ug}.$$

In a symmetric equilibrium:

$$p_{i_g} = p_g$$
$$y_{i_g} = y_g$$
$$m_{i_g} = m_g$$

and

$$m_g N_g = M_g.$$

We also have

$$M_g = Y_g.$$

Factor shares in each industry are

$$\begin{aligned}
S_{LYg} &:= \sum_{j=1}^{J} \frac{W_j L_{jg,Y}}{p_g Y_g} = \gamma_{Yg} \frac{1}{\mu_g} \\
S_{LNg} &:= \sum_{j=1}^{J} \frac{W_j L_{jg,N}}{p_g Y_g} = \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right) \\
S_{Lg} &:= \gamma_{Yg} \frac{1}{\mu_g} + \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right).
\end{aligned}$$

Note – these are shares in the industry value added, not in the aggregate nominal GDP. The aggregate labor share is

$$\begin{aligned}
S_L &:= \frac{\sum_{g=1}^{G} \sum_{j=1}^{J} W_j \left(L_{jg,Y} + L_{jg,N}\right)}{pY} \\
&= \frac{\sum_{g=1}^{G} p_g Y_g \frac{\sum_{j=1}^{J} W_j \left(L_{jg,Y} + L_{jg,N}\right)}{p_g Y_g}}{pY} \\
&= \sum_{g=1}^{G} \frac{p_g Y_g}{pY} S_{Lg} \\
&= \sum_{g=1}^{G} \varsigma_g S_{Lg}
\end{aligned}$$

The income share of occupation j in industry g is

$$\begin{aligned}
S_{jg} &:= \frac{W_j L_{jg}}{p_g Y_g} \\
&= \eta_{jgY} \times \gamma_{Yg} \frac{1}{\mu_g} + \eta_{jgN} \times \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right) \\
&= \eta_{jgY} S_{LYg} + \eta_{jgN} S_{LNg}
\end{aligned}$$

so we can write $s_{jg} := S_{jg}/S_{Lg}$ as

$$\begin{aligned}
s_{jg} &:= \frac{W_j L_{jg}}{p_g Y_g} \times \left(\frac{\sum_{j=1}^{J} W_j L_{jg}}{p_g Y_g} \right)^{-1} \\
&= \eta_{jgY} \frac{S_{LYg}}{S_{Lg}} + \eta_{jgN} \frac{S_{LNg}}{S_{Lg}} \\
&= \eta_{jgY} + \left(\eta_{jgN} - \eta_{jgY}\right) \frac{S_{LNg}}{S_{Lg}} \\
&= \eta_{jgY} + \left(\eta_{jgN} - \eta_{jgY}\right) \frac{\gamma_{Ng}}{\gamma_{Ng} - \gamma_{Yg}} \left(1 - \frac{\gamma_{Yg}}{S_{Lg}}\right)
\end{aligned}$$

The aggregate income share of occupation j is

$$\begin{aligned} S_j &:= \frac{W_j L_j}{pY} \\ &= \frac{\sum_{g=1}^{G} W_j \left(L_{jg,Y} + L_{jg,N}\right)}{pY} \\ &= \sum_{g=1}^{G} \frac{W_j \left(L_{jg,Y} + L_{jg,N}\right)}{p_g Y_g} \frac{p_g Y_g}{pY} \\ &= \sum_{g=1}^{G} \varsigma_g S_{jg} \end{aligned}$$

and $s_j := S_j/S_L$ is

$$\begin{aligned} s_j &:= \frac{W_j L_j}{pY} \times \left(\frac{\sum_{j=1}^{J} W_j L_j}{pY}\right)^{-1} \\ &= \sum_{g=1}^{G} \varsigma_g \frac{S_{jg}}{S_L} \end{aligned}$$

or, alternatively,

$$s_j = \sum_{g=1}^{G} s_{jg} \varsigma_g \frac{S_{Lg}}{S_L}.$$

$N-$intensity of occupation j is defined as

$$\begin{aligned} \frac{S_{LNj}}{S_{Lj}} &:= \frac{\sum_{g=1}^{G} W_j L_{jg,N}}{\sum_{g=1}^{G} W_j \left(L_{jg,N} + L_{jg,Y}\right)} \\ &= \frac{\sum_{g=1}^{G} \varsigma_g \left(\eta_{jgN} \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right)\right)}{\sum_{g=1}^{G} \varsigma_g \left[\eta_{jgN} \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right) + \eta_{jgY} \gamma_{Yg} \frac{1}{\mu_g}\right]} \end{aligned}$$

What is the mapping between the model with industries and the baseline model shown in Section 2? In an equilibrium the aggregate output is

$$\begin{aligned} Y &= \prod_{g=1}^{G} Y_g^{\varsigma_g} \\ &= \prod_{g=1}^{G} \left(Z_{Ug} \left(\prod_{j=1}^{J} L_{jYg}^{\varsigma_g \eta_{jgY} \gamma_{Yg}}\right)\right). \end{aligned}$$

We can define

$$\omega_{Yg} := \gamma_{Yg}\varsigma_g$$
$$\gamma_Y := \sum_{g=1}^{G} \omega_{Yg}$$
$$\eta_{jY} := \gamma_Y^{-1}\sum_{g=1}^{G} \omega_{Yg}\eta_{jgY}$$

to rewrite the aggregate output as

$$Y = \left(\prod_{g=1}^{G} Z_{Yg}^{\varsigma_g}\right) \times \left(\prod_{j=1}^{J}\left(\prod_{g=1}^{G} L_{jYg}^{\eta_{jgY}\omega_{Yg}}\right)\right).$$

To to understand how $L_{jg,Y}$ is related to $L_{jY} := \sum_{g=1}^{G} L_{jg,Y}$ use the upstream first order conditions for a pair of industries (g,s). They imply

$$\frac{P_{Ug}\gamma_{Yg}}{P_{Us}\gamma_{Ys}}\frac{M_{Ug}}{M_{Us}} \times \frac{\eta_{jgY}}{\eta_{jsY}} L_{js,Y} = L_{jg,Y}.$$

Use $M_{Ug} = Y_{Ug} = \varsigma_g \left(\frac{p_g}{p}\right)^{-1} Y$ to write

$$\frac{P_{Ug}\gamma_{Yg}}{P_{Us}\gamma_{Ys}}\frac{\varsigma_g\left(\frac{p_g}{p}\right)^{-1}}{\varsigma_s\left(\frac{p_s}{p}\right)^{-1}} \times \frac{\eta_{jgY}}{\eta_{jsY}} L_{js,Y} = L_{jg,Y}$$

so

$$\frac{\frac{P_{Ug}}{p_g}\omega_{Yg}}{\frac{P_{Us}}{p_s}\omega_{Ys}} \times \frac{\eta_{jgY}}{\eta_{jsY}} L_{js,Y} = L_{jg,Y}$$

but $\frac{P_{Ug}}{p_g} = \frac{1}{\mu_g}$, therefore

$$\frac{\frac{1}{\mu_g}\omega_{Yg}}{\frac{1}{\mu_s}\omega_{Ys}}\frac{\eta_{jgY}}{\eta_{jsY}} L_{js,Y} = L_{jg,Y}$$

We can then write

$$L_{js,Y} = \frac{\frac{1}{\mu_s}\omega_{Ys}\eta_{jsY}}{\sum_{g=1}^{G}\frac{1}{\mu_g}\omega_{Yg}\eta_{jgY}} L_{jY}$$

i.e.

$$L_{jYs} = \mathcal{F}_{Ys}L_{jY}$$
$$\mathcal{F}_{Ys} := \frac{\frac{1}{\mu_s}\omega_s\eta_{jsY}}{\sum_{g=1}^{G}\frac{1}{\mu_g}\omega_g\eta_{jgY}}$$

This allows us to write

$$Y = Z_U \times \left(\prod_{j=1}^{J} L_{jY}^{\eta_{jY}} \right)^{\gamma_Y}$$
$$Z_U := \left(\prod_{g=1}^{G} Z_{Ug}^{\varsigma_g} \mathcal{F}_{Yg}^{\omega_{Yg}} \right)$$

which looks like the production function in our baseline model (but now Z_Y depends on the distribution of markups). What is S_{LY}? Recall that in a model with multiple industries

$$S_{LYg} = \gamma_{Yg} \frac{1}{\mu_g}$$

and

$$S_{LY} := \frac{\sum_{g=1}^{G} \sum_{j=1}^{J} W_j L_{jg,Y}}{pY}$$
$$= \sum_{g=1}^{G} \varsigma_g \gamma_{Yg} \frac{1}{\mu_g}$$

Define the aggregate markup

$$\frac{1}{\mu} := \left(\sum_{g=1}^{G} \varsigma_g \frac{1}{\mu_g} \right).$$

We have

$$S_{LY} = \gamma_Y \frac{1}{\mu} + Cov\left[\gamma_{Yg}, \frac{1}{\mu_g} \right]$$

Define

$$N := \prod_{g=1}^{G} N_g^{\varsigma_g}$$

and define

$$\Pi := \prod_{g=1}^{G} \left(\frac{\left(1 - \frac{1}{\mu}\right)}{\left(1 - \frac{1}{\mu_g}\right)} \frac{\Pi_g}{\varsigma_g} \right)^{\varsigma_g}$$

where

$$\frac{1}{\mu} = \sum_{g=1}^{G} \varsigma_g \frac{1}{\mu_g}$$

This definition of Π gives us

$$N\Pi = \left(1 - \frac{1}{\mu}\right) pY.$$

Now define

$$\omega_{Ng} := \gamma_{Ng}\varsigma_g$$
$$\gamma_N := \sum_{g=1}^{G} \omega_{Ng}$$
$$\eta_{jN} := \gamma_N^{-1} \sum_{g=1}^{G} \omega_{Ng}\eta_{jgN}$$

we can use the definition of N together with downstream first order conditions for a pair of industries (g, s) to derive

$$N = Z_D \left(\prod_{j=1}^{J} L_{jN}^{\eta_{jN}} \right)^{\gamma_N}$$

where

$$Z_D := \left(\prod_{g=1}^{G} Z_{Dg}^{\varsigma_g} \mathcal{F}_{Ng}^{\omega_{Ng}} \right)$$
$$\mathcal{F}_{Ns} := \frac{\left(1 - \frac{1}{\mu_s}\right) \omega_{Ns}\eta_{jsN}}{\sum_{g=1}^{G} \left(1 - \frac{1}{\mu_g}\right) \omega_{Ng}\eta_{jgN}}$$

What is S_{LN}? Recall we had

$$S_{LNg} = \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right)$$

$$S_{LN} := \frac{\sum_{g=1}^{G} \sum_{j=1}^{J} W_j L_{jg,N}}{pY}$$
$$= \sum_{g=1}^{G} \varsigma_g \gamma_{Ng} \left(1 - \frac{1}{\mu_g}\right)$$

and we have

$$S_{LN} = \gamma_N \left(1 - \frac{1}{\mu}\right) + Cov\left[\gamma_{Ng}, \left(1 - \frac{1}{\mu_g}\right)\right]$$

We can thus see that if

$$Cov\left[\gamma_{Ng}, \left(1 - \frac{1}{\mu_g}\right)\right] = Cov\left[\gamma_{Yg}, \frac{1}{\mu_g}\right] = 0$$

we have

$$S_L = \gamma_N \left(1 - \frac{1}{\mu}\right) + \gamma_Y \frac{1}{\mu}$$

where

$$\gamma_N = \sum_{g=1}^{G} \gamma_{Ng} \varsigma_g$$

$$\gamma_Y = \sum_{g=1}^{G} \gamma_{Yg} \varsigma_g$$

$$\frac{1}{\mu} = \sum_{g=1}^{G} \frac{1}{\mu_g} \varsigma_g.$$

The expressions for occupational shares are

$$\begin{aligned} s_j &= \sum_{g=1}^{G} \varsigma_g \frac{S_{Lg}}{S_L} s_{jg} \\ &= \sum_{g=1}^{G} \left(\eta_{jgY} \frac{\varsigma_g S_{Lg}}{\sum_{g=1}^{G} \varsigma_g S_{Lg}} + \varsigma_g \left(\eta_{jgN} - \eta_{jgY} \right) \frac{\gamma_{Ng}}{\gamma_{Ng} - \gamma_{Yg}} \left(\frac{\varsigma_g S_{Lg}}{\sum_{g=1}^{G} \varsigma_g S_{Lg}} - \frac{\gamma_{Yg}}{\sum_{g=1}^{G} \varsigma_g S_{Lg}} \right) \right). \end{aligned}$$

In a special case with $\frac{1}{\mu_g} = \frac{1}{\mu}$ it can be rewritten as

$$s_j = \eta_{jY} \frac{\frac{1}{\mu}\gamma_Y}{\frac{1}{\mu}\gamma_Y + \left(1 - \frac{1}{\mu}\right)\gamma_N} + \eta_{jN} \frac{\left(1 - \frac{1}{\mu}\right)\gamma_N}{\frac{1}{\mu}\gamma_Y + \left(1 - \frac{1}{\mu}\right)\gamma_N}$$

which is the same as in the baseline model.

E Details of Data and Additional Estimation Results

E.1 Detailed Data Description

Labor share

- **Baseline Gomme and Rupert:** The measure excludes the household and government sectors and uses NIPA tables 1.12 and 1.7.5 and corresponds to the second alternative measure of the labor share proposed in Gomme and Rupert (2004). They define unambiguous labor income as compensation of employees, and unambiguous capital income (as corporate profits, rental income, net interest income, and depreciation. The remaining (ambiguous) components are then proprietors' income plus indirect taxes net of subsidies (NIPA Table 1.12). These are apportioned to capital and labor in the same proportion as the unambiguous components. Here CE_t is compensation of employees (line 2 in NIPA table 1.12), RI_t rental income (line 12 in NIPA table 1.12), CP_t corporate profits before tax (line 13 in NIPA table 1.12), NI_t net interest income (line 18 in NIPA table 1.12) and δ_t depreciation (line 5 in table 1.7.5) .
$$LS_t = \frac{CE_t}{CE_t + RI_t + CP_t + NI_t + \delta_t}$$
- **BLS Nonfarm Business:** U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Labor Share (FRED id: PRS85006173)

- **BLS Nonfinancial:** U.S. Bureau of Labor Statistics, Nonfinancial Corporations Sector: Labor Share (FRED id: PRS88003173)
- **Cooley and Prescott:** Follows Cooley and Prescott (1995). The labor share of income is defined as one minus capital income divided by output. Cooley and Prescott assume that the proportion of ambiguous capital income ACI_t to ambiguous income AI_t is the same as the proportion of unambiguous capital income to unambiguous income. Ambiguous income, AI_t is the sum of proprietors income (line 9, NIPA table 1.12), taxes on production less subsidies (lines 19 and 20, NIPA Table 1.12), business current transfer payments (line 21, NIPA Table 1.12) . Unambiguous income UI_t consists of compensation of employees (line 2 in NIPA table 1.12) and unambiguous capital income UCI_t which in turn consists of rental income (line 12, NIPA Table 1.12), net interests (line 13, Table 1.12), corporate profits (line 18, NIPA Table 1.12) and current surplus of government enterprises (line 25, NIPA Table 1.12). Formally

$$
\begin{aligned}
CS_t^U &= \frac{UCI_t + \delta_t}{UI_t} \\
ACI_t &= CS_t^U AI_t \\
LS_t &= 1 - \frac{UCI_t + \delta_t + ACI_t}{GNP_t}
\end{aligned}
$$

  where δ_t is depreciation (line 5 in table 1.7.5)
- **Fernald:** It is taken from Fernald (2014). It is utilization adjusted quarterly series.

Markup

- Data from 1950 Q1 to 2019 Q4 from U.S. Bureau of Labor Statistics. We use the following series:
  - **WPSFD49207**, Producer Price Index by Commodity for Final Demand: Finished Goods, Seasonally Adjusted
  - **WPSID61**, Producer Price Index by Commodity: Intermediate Demand by Commodity Type: Processed Goods for Intermediate Demand

Occupational income shares

- We use data from the monthly Outgoing Rotation Groups of the Current Population Survey (CPS-ORG) to construct quarterly series for occupational income shares s_j . We restrict attention to employed individuals aged 16 and over not living in group quarters, and measure labor income with the IPUMS variable 'earnweek'. This variable reports the amount (in dollars) a given individual earned from their job each week before deductions.
- We compute quarterly labor income by summing weekly labor income for individuals in each of 9 broad occupation categories, which we construct from the 389 OCC1990 occupation codes in a following way:
  - Managerial occupations: OCC1990 codes 3 - 37
  - Professional specialty occupations: OCC1990 codes 43 - 200
  - High-tech occupations: OCC1990 codes 203 - 235
  - Sales occupations: OCC1990 codes 243 - 283
  - Administrative support and clerical occupations: OCC1990 codes 303 - 389
  - Service occupations OCC1990 codes 405 - 469

- Farming, forestry, and fishing occupations, construction and extractive occupations: OCC1990 codes 473-498, 558 - 599 and 614 - 617
  - Precision production occupations and repair: OCC1990 codes 503 - 549 and 628 - 699
  - Machine operators, assemblers, inspectors, transportation and material moving occupations: OCC1990 codes1990 703 - 799 and 803 - 889

- We remove all observations with OCC1990 codes that do not belong to any of the above 9 broad categories, for example observations with missing OCC1990 codes or military occupations. We then divide the sum of labor income of individuals in each broad category by the sum of labor income of all individuals in any given quarter to obtain s_j.
- Because there was a change in occupational codes used in CPS-ORG in 2002 we make the following adjustment: we interpolate a difference between s_j in 2002Q4 and s_j in 2003Q1 and then we recalculate s_j by adding a cumulative difference to the first value.. This assumes that all changes in s_j between 2002 and 2003 were due to the change in occupational codes.
- We then use X-12-ARIMA to do seasonal adjustment of s_j and renormalize s_j so that their sum in each quarter is always equal to 1.

Occupational total hours and median hourly wages

- We use data from the 1980 Census and 2015 American Community Survey (ACS) to calculate occupational total hours and median hourly wages. We restrict attention to employed individuals aged 18 to 65 and not living in group quarters, and measure labor income with the IPUMS variable 'incwage'. This variable reports the amount (in dollars) of each respondent's total pre-tax wage and salary income - that is, money received as an employee - for the previous 12 months. Sources of income in 'incwage' include wages, salaries, commissions, cash bonuses, tips, and other money income received from an employer. Payments-in-kind or reimbursements for business expenses are not included.
- We keep only individuals with positive and known wage income and hours and positive weights
- To calculate the number of hours per year that the respondent usually worked we use variables 'uhrswork' and 'wkswork2'. 'uhrswork' reports the number of hours per week that the respondent usually worked, if the person worked during the previous 12 months. 'wkswork2' reports the number of weeks that the respondent worked for profit, pay, or as an unpaid family worker during the previous 12 months. Because 'wkswork2' is reported in intervals (1-13 weeks, 14-26 weeks, and so on), instead of the precise number of weeks, we associate each value of 'wkswork2' with the midpoint of the corresponding interval (for example if 'wkswork2'=1 we treat it as 7 weeks). We multiply 'uhrswork' by our measure of weeks based on 'wkswork2'. Hourly wages are then computed by dividing 'incwage' by the number of hours per year.
- We calculate occupational total hours by summing yearly hours for individuals in each of 9 broad occupation categories.

Downstream processing

- Average Hourly Compensation in the Non-Farm Business Sector, from the BLS (PRS85006103) is provided as an index (equal to 100 in 2012).
- Average Hourly Earnings of Production and Nonsupervisory Employees, Total Private, from the BLS (CES0500000008) is expressed in dollars per hour. Production and related employees include working supervisors and all nonsupervisory employees. Nonsupervisory employees include those individuals in private, service-providing industries who are not above the working-supervisor level.

	(1) Baseline Gomme-Rupert	(2) BLS Non-farm Business	(4) Cooley-Prescott	(4) Fernald	(5) BLS Non-financial
γ_Y	0.620	0.602	0.621	0.646	0.594
	(0.005)	(0.006)	(0.007)	(0.007)	(0.008)
γ_N	0.804	0.699	0.788	0.780	0.778
	(0.025)	(0.020)	(0.032)	(0.035)	(0.039)
Implied value of $\frac{S_{LN}}{S_L}$	21%	19%	20%	19%	21%
P-val for test $\gamma_N = \gamma_Y$	0.00	0.01	0.00	0.00	0.00
Assumed mean markup, μ	1.2	1.2	1.2	1.2	1,2
Mean S_L	65%	62%	65%	67%	63%

Table 6: First step estimation results: alternate labor share series

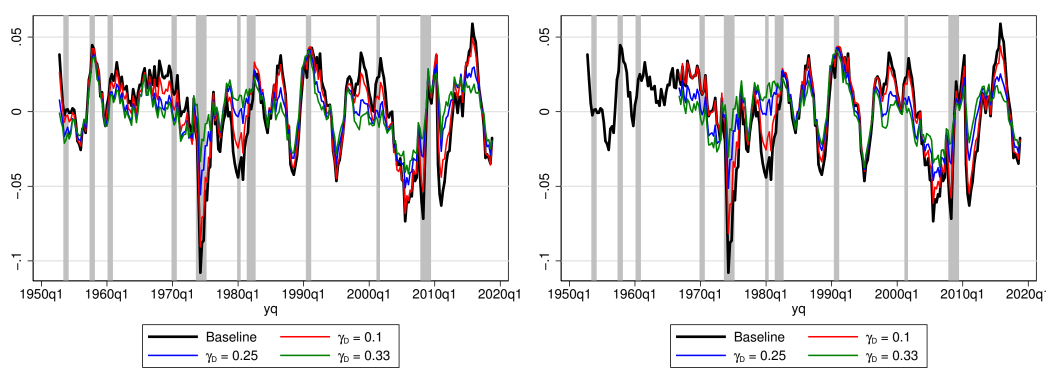
Model with industries

- We use BEA KLEMS data and WORLD KLEMS data. First, we map 63 sectors in BEA KLEMS data to 11 NAICS super-sectors. We sum up labor compensation and value added of all sectors belonging to a super-sector. We exclude super-sector 11 (government) and calculate total value added and total labor compensation in each year. We calculate labor share of a super-sector by dividing its labor compensation by its value added. We calculate value added share of a supersector by dividing its value added by the total value added in each year. We perform seasonal adjustment using X-12-ARIMA and de-trend data with the Hamilton filter. After that we add sample means of labor shares and value added shares to de-trended data. Finally, we re-normalize value added shares to ensure that they sum up to one in every year.

E.2 Additional Tables and Figures

	(1) Baseline Hamilton	(2) Baxter-King	(3) Christiano-Fitzgerald	(4) Linear Trend	(5) Quadratic Trend	(6) Hodrick-Prescott
γ_Y	0.620	0.634	0.634	0.623	0.619	0.630
	(0.005)	(0.006)	(0.005)	(0.004)	(0.004)	(0.006)
γ_N	0.804	0.734	0.733	0.785	0.810	0.750
	(0.025)	(0.028)	(0.025)	(0.021)	(0.018)	(0.028)
Implied value of $\frac{S_{LN}}{S_L}$	21%	19%	19%	20%	21%	19%
P-val for test $\gamma_N = \gamma_Y$	0.00	0.00	0.00	0.00	0.00	0.00
Assumed mean markup, μ	1.2	1.2	1.2	1.2	1.2	1.2

Table 7: First step estimation results: alternate methods of de-trending



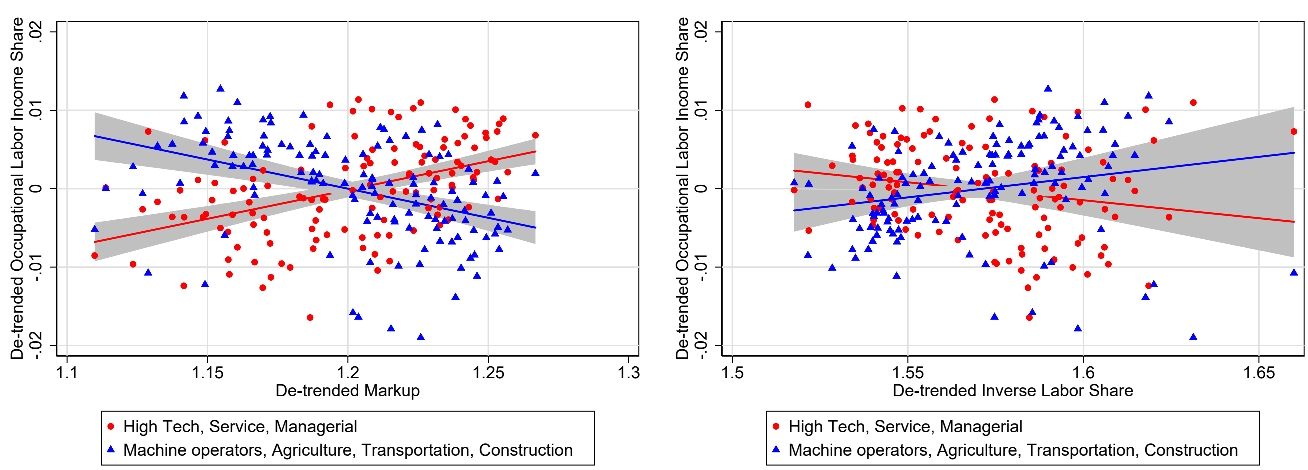
(a) W_Y: Non-farm Business Sector: Compensation Per Hour

(b) W_Y: Average Hourly Earnings of Production and Nonsupervisory Employees

Figure 4: Cyclical components of alternative markup series:

	(1) Baseline	(2) GDP	(3) GDP + 4 lags	(4) GDP + 4 + interactions
γ_Y	0.620	0.620	0.620	0.618
	(0.005)	(0.005)	(0.004)	(0.004)
γ_N	0.804	0.805	0.802	0.810
	(0.025)	(0.025)	(0.019)	(0.021)
GDP_t		0.011	-0.188	1.351
		(0.018)	(0.034)	(1.337)
GDP_{t-1}		.	0.084	-1.048
			(0.048)	(2.128)
$GDP_t \times \frac{1}{\mu_t}$				-1.846
				(1.600)
Implied value of $\frac{S_{LN}}{S_L}$	21%	20%	20%	20%
P-val for test $\gamma_N = \gamma_Y$	0.00	0.00	0.00	0.00
Assumed mean markup, μ	1.2	1.2	1.2	1.2

Table 8: First step estimation results: controlling for GDP. GDP is measured as a cyclical component of log real GDP per capita, detrended using the Hamilton filter. Seasonally adjusted real GDP per capita series are from BEA (A939RX). In column (2) we control for contemporaneous GDP, in column (3) for contemporaneous and 4 lags, in column (4) we have contemporaneous GDP and 4 lags and their interactions with the measure of markups.



(a) Reduced form scatter plot

(b) Raw scatter plot, without instrument

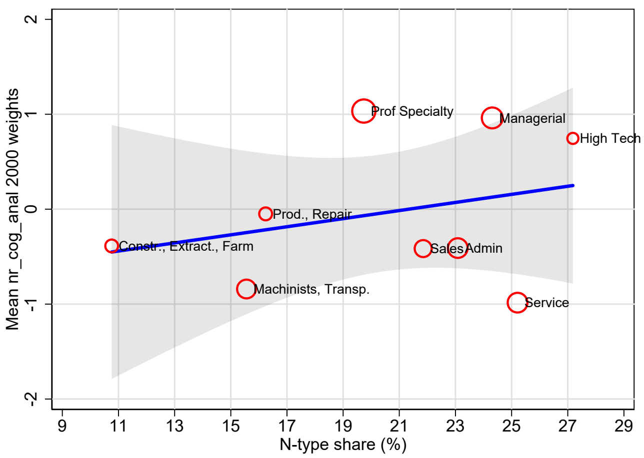
Figure 5: Cyclical components of occupational income shares

		η_Y	η_N	P-val $\eta_Y = \eta_N$	Elasticity ε_{S_j,S_L}	Share $\frac{S_{jN}}{S_j}$
Panel A: $\mu = 1.05$						
High Tech Occs		0.042	0.057	0.016	2.68	25%
Service Occs		0.078	0.096	0.005	2.17	24%
Managerial Occs		0.205	0.245	0.000	1.94	23%
Admin Support, Clerical		0.114	0.128	0.108	1.63	22%
Sales Occs		0.098	0.104	0.378	1.32	21%
Professional Specialty		0.221	0.211	0.330	0.78	19%
Production, Repair		0.068	0.053	0.002	-0.11	16%
Machine Operators, Transportation		0.119	0.088	0.000	-0.28	16%
Construction, Extractive, Farming		0.055	0.028	0.001	-1.49	11%
First stage: R2	0.27					
First stage F	37.6					
Panel B: $\mu = 1.35$						
High Tech Occs		0.037	0.057	0.016	2.68	30%
Service Occs		0.070	0.096	0.005	2.17	27%
Managerial Occs		0.190	0.245	0.000	1.94	26%
Admin Support, Clerical		0.108	0.128	0.108	1.63	24%
Sales Occs		0.095	0.104	0.378	1.32	23%
Professional Specialty		0.225	0.211	0.330	0.78	20%
Production, Repair		0.074	0.053	0.002	-0.11	16%
Machine Operators, Transportation		0.131	0.088	0.000	-0.28	15%
Construction, Extractive, Farming		0.065	0.028	0.001	-1.49	10%
First stage: R2	0.27					
First stage F	37.6					

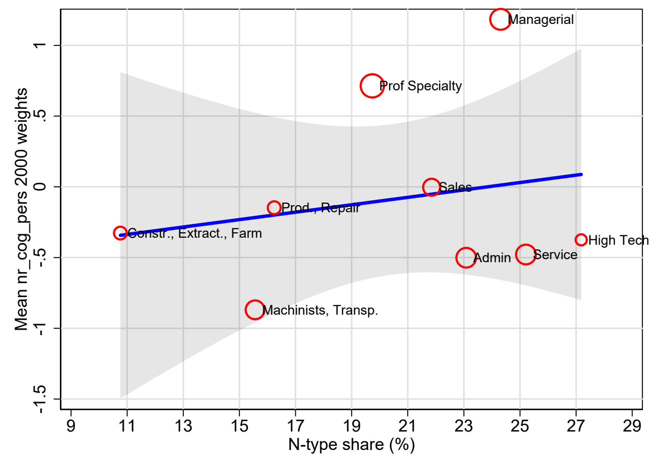
Table 9: Second step estimates of occupational factor share parameters. Estimates use de-trended markup as an instrument for de-trended inverse labor share.

				P-val	Elasticity	Share
		η_Y	η_N	$\eta_Y = \eta_N$	ε_{S_j,S_L}	$\frac{S_{jN}}{S_j}$
Panel A: $\gamma_D = 0.10$						
High Tech Occs		0.037	0.065	0.007	3.29	32%
Service Occs		0.071	0.108	0.001	2.66	29%
Managerial Occs		0.194	0.256	0.000	2.07	27%
Admin Support, Clerical		0.109	0.134	0.077	1.76	25%
Sales Occs		0.098	0.098	1.000	1.00	21%
Professional Specialty		0.222	0.216	0.676	0.89	21%
Production, Repair		0.072	0.050	0.003	-0.17	16%
Machine Operators, Transportation		0.128	0.079	0.001	-0.47	14%
Construction, Extractive, Farming		0.064	0.014	0.000	-2.30	6%
First stage: R2	0.21					
First stage F	33.2					
Panel B: $\gamma_D = 0.25$						
High Tech Occs		0.034	0.075	0.009	4.55	37%
Service Occs		0.067	0.123	0.001	3.67	33%
Managerial Occs		0.190	0.271	0.002	2.45	28%
Admin Support, Clerical		0.109	0.134	0.153	1.81	25%
Sales Occs		0.101	0.087	0.366	0.46	19%
Professional Specialty		0.218	0.232	0.509	1.23	22%
Production, Repair		0.073	0.045	0.007	-0.57	14%
Machine Operators, Transportation		0.130	0.071	0.005	-0.88	13%
Construction, Extractive, Farming			0.000			0%
First stage: R2	0.10					
First stage F	19.2					
Panel C: $\gamma_D = \gamma_Y = 0.37$						
High Tech Occs		0.017	0.133	0.220	10.14	69%
Service Occs		0.040	0.213	0.174	8.41	60%
Managerial Occs		0.161	0.368	0.183	4.36	39%
Admin Support, Clerical		0.111	0.127	0.735	1.48	24%
Sales Occs		0.117	0.034	0.351	-1.84	7%
Professional Specialty		0.192	0.320	0.258	2.93	32%
Production, Repair		0.083	0.013	0.187	-2.49	4%
Machine Operators, Transportation		0.146	0.018	0.225	-2.68	3%
Construction, Extractive, Farming			0.000			0%
First stage: R2	0.005					
First stage F	1.84					

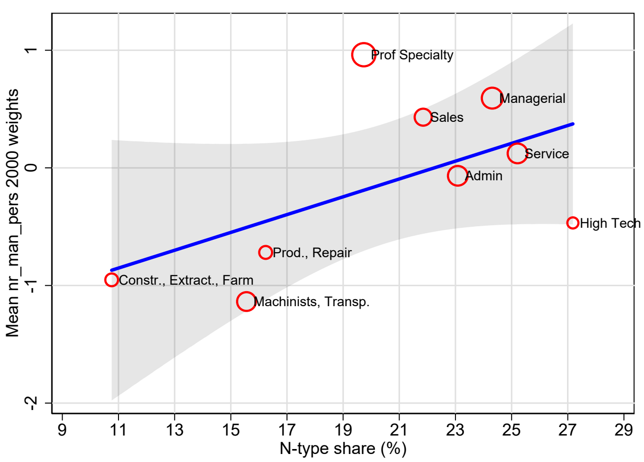
Table 10: Second step estimates of occupational factor share parameters with further downstream processing ($\gamma_D > 0$). Estimates use de-trended markup (calculated using the average real wages and the ratio of price indices and nominal wage) as an instrument for de-trended inverse labor share.



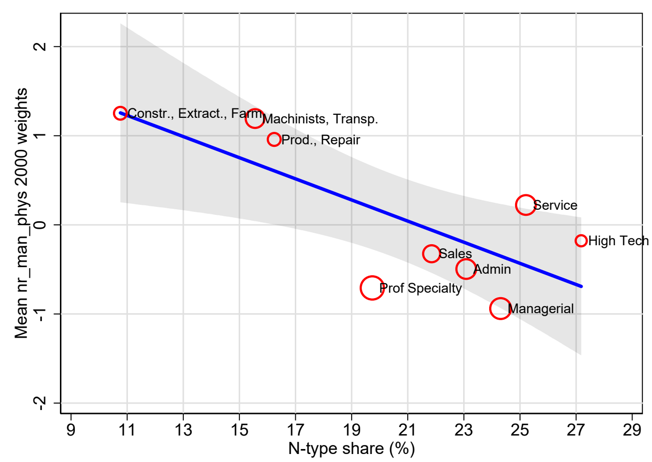
(a) Non-routine, cognitive, analytical



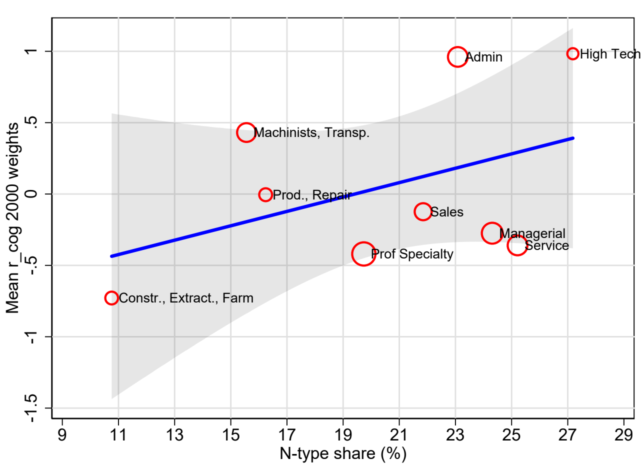
(b) Non-routine, cognitive, inter-personal



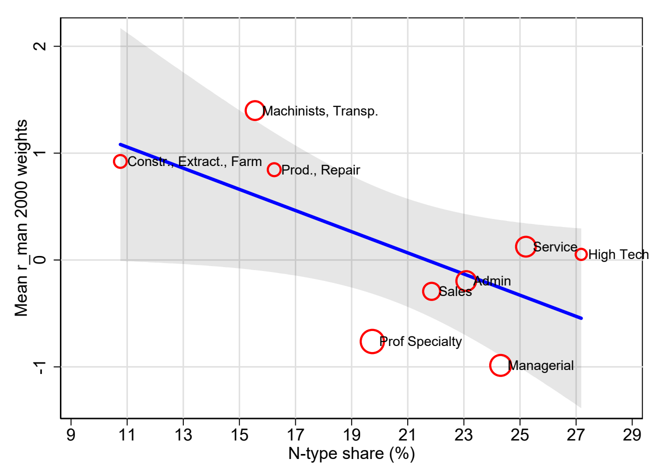
(c) Non-routine, inter-personal, manual



(d) Non-routine, physical, manual



(e) Routine, cognitive



(f) Routine, manual

Figure 6: Correlation of N-content of occupations with other occupation characteristics

	γ_{Yg}	γ_{Ng}	P-val test for $\gamma_{Yg} = \gamma_{Ng}$	Implied $\frac{S_{LNg}}{S_{Lg}}$	Mean S_{Lg}	Mean share of value added
Resources and Mining	0.196	1.896	0.000	65.9%	47.7%	3.1%
Construction	0.803	1.054	0.000	20.8%	84.5%	4.8%
Manufacturing	0.512	0.796	0.000	23.7%	55.9%	16.3%
Trade, Transp., Util.	0.534	0.729	0.000	21.4%	56.7%	19.9%
Information	0.373	0.588	0.137	24.0%	40.8%	5.6%
Finance	0.259	0.239	0.860	15.6%	25.5%	22.3%
Prof. and Business Serv.	0.795	0.960	0.080	19.4%	82.2%	12.3%
Educ. and Health Serv.	0.876	0.948	0.315	17.8%	88.8%	8.7%
Leisure and Hospitality	0.678	0.747	0.359	18.1%	69.0%	4.3%
Other Services	0.785	0.847	0.699	17.8%	79.5%	2.8%

Table 11: First step estimation results, model with industries, no restrictions on $(\gamma_{Yg}, \gamma_{Ng})$ BEA KLEMS annual data 1987-2019. Assumed mean markup, μ=1.2

	γ_{Yg}	γ_{Ng}	P-val test for $\gamma_{Yg} = \gamma_{Ng}$	Implied $\frac{S_{LNg}}{S_{Lg}}$	Mean S_{Lg}	Mean share of value added
Resources and Mining	0.312	1.000	0.000	39.1%	42.4%	5.5%
Construction	0.875	1.000	0.000	18.6%	89.6%	5.2%
Manufacturing	0.586	0.762	0.139	20.6%	61.5%	23.9%
Trade, Transp., Util.	0.649	0.819	0.124	20.1%	67.7%	20.5%
Information	0.421	0.727	0.021	25.7%	47.2%	4.8%
Finance	0.227	0.209	0.718	15.5%	22.4%	19.3%
Prof. and Business Serv.	0.585	0.671	0.409	18.6%	60.0%	8.4%
Educ. and Health Serv.	0.766	1.000	0.146	20.7%	80.5%	5.9%
Leisure and Hospitality	0.757	0.720	0.773	16.0%	75.1%	3.6%
Other Services	0.832	1.000	0.000	19.4%	85.9%	3.0%

Table 12: First step estimation results, with industries, $(\gamma_{Yg}, \gamma_{Ng})$ restricted to be between 0 and 1. WORLD KLEMS annual data 1950-2014. Assumed mean markup, μ=1.2

F Trends

Let

$$S_{L,t} = \gamma_{N,t} + \left(\gamma_{Y,t} - \gamma_{N,t}\right) \frac{1}{\mu_t} + \epsilon_{L,t}$$

and assume deterministic trends

$$\begin{aligned}
\frac{1}{\mu_t} &= g_\mu\left(\beta_\mu, t\right) + \epsilon_{\mu,t} \\
\gamma_{N,t} &= g_\gamma\left(\beta_\gamma, t\right) + \gamma_N \\
\gamma_{Y.t} &= g_\gamma\left(\beta_\gamma, t\right) + \gamma_Y
\end{aligned}$$

where $E\left[\epsilon_{\mu,t}\right] = 0$ and $E\left[\epsilon_{L,t}\right] = 0$. Note common trend in $\gamma_{N,t}$ and $\gamma_{Y,t}$. We have

$$\begin{aligned}
S_{L,t} &= \gamma_N + g_\gamma\left(\beta_\gamma, t\right) + \left(\gamma_Y - \gamma_N\right)\left[g_\mu\left(\beta_\mu, t\right) + \epsilon_{\mu,t}\right] + \epsilon_{L,t} \\
&= \gamma_N + g_\gamma\left(\beta_\gamma, t\right) + \left(\gamma_Y - \gamma_N\right) g_\mu\left(\beta_\mu, t\right) + \left(\gamma_Y - \gamma_N\right)\epsilon_{\mu,t} + \epsilon_{L,t}
\end{aligned}$$

which can be written as

$$\begin{aligned}
S_{L,t.} &= g_{S_L}\left(\beta_{S_L}, t\right) + \hat{S}_{L,t} \\
g_{S_L}\left(\beta_{S_L}, t\right) &:= \gamma_N + g_\gamma\left(\beta_\gamma, t\right) + \left(\gamma_Y - \gamma_N\right) g_\mu\left(\beta_\mu, t\right) \\
\hat{S}_{L,t} &:= \left(\gamma_Y - \gamma_N\right)\epsilon_{\mu,t} + \epsilon_{L,t}
\end{aligned}$$

Regression of $\hat{S}_{Lt}$ (de-trended S_{Lt}), on $\epsilon_{\mu,t}$ (de-trended $\frac{1}{\mu_t}$) allows us to recover $\left(\gamma_Y - \gamma_N\right)$ if

$$\begin{aligned}
E\left[\epsilon_{L,t}\right] &= 0 \ \forall t \\
E\left[\epsilon_{L,\tau} \middle| \epsilon_{\mu,t}\right] &= 0 \ \forall\left(t, \tau\right).
\end{aligned}$$

Let $\bar{x}$ be the sample average of variable x_r. Let $\overline{g_\mu\left(\beta_\mu, t\right)} := \frac{1}{\mu}$. We then have

$$\begin{aligned}
\bar{S}_L &= \overline{g_{S_L}\left(\beta_{S_L}, t\right)} \\
&= \gamma_N + \overline{g_\gamma\left(\beta_\gamma, t\right)} + \left(\gamma_Y - \gamma_N\right)\overline{g_\mu\left(\beta_\mu, t\right)} \\
&= \gamma_N + \overline{g_\gamma\left(\beta_\gamma, t\right)} + \left(\gamma_Y - \gamma_N\right)\frac{1}{\mu}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{S}_{L,t} &:= \hat{S}_{L,t} + \bar{S}_L \\
&= \gamma_N + \overline{g_\gamma\left(\beta_\gamma, t\right)} + \left(\gamma_Y - \gamma_N\right)\left(\frac{1}{\mu} + \epsilon_{\mu t}\right) + \epsilon_{L,t} \\
&= \gamma_N + \overline{g_\gamma\left(\beta_\gamma, t\right)} + \left(\gamma_Y - \gamma_N\right)\left(\frac{\tilde{1}}{\mu_t}\right) + \epsilon_{L,t}
\end{aligned}$$

We normalize $\overline{g_\gamma\left(\beta_\gamma, t\right)} = 0$ and obtain

$$\tilde{S}_{L,t} = \gamma_N + \left(\gamma_Y - \gamma_N\right)\left(\frac{\tilde{1}}{\mu_t}\right) + \epsilon_{L,t}$$

which corresponds to equation (2).